

# State Feedback Control of DC-DC Converter Using LQR Integral Controller and Kalman Filter Observer



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**Abstract** In this paper, the linear state feedback control using LQR controller for a DC/DC converter in the case of negative voltages topology is presented in order to achieve a particular desired behavior. To guarantee a zero steady-state error, we introduce an integral action, which will work out this problem by assuring that the steady-state error will end up to zero. For filtering and state estimation with a low cost and less complexity a state observer is obtained based a Kalman Filter observer. Detailed simulation study is presented to demonstrating the robustness and effectiveness of the proposed control scheme.

**Keywords** Linear quadratic regulator · DC/DC Buck-Boost Converter · Kalman filter · Static error

## 1 Introduction

Power supply technology enables technologies that allow us to operate circuits and electronic systems. All digital and analog circuits require a power source to operate. Many of these circuits require multiple DC supply voltages. DC power supplies

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are widely used in telecommunications, household appliances, defense, and medical electronics [1]. The DC voltage is generally obtained from a battery or by converting AC current into DC current using a transformer, rectifier, and filter [1, 2].

In some power supplies, a negative output voltage are required. There are many different methods for generating a negative voltage in the output from a positive voltage in the input. A very simple and inexpensive method for obtaining a negative voltage is the Buck-Boost converter. The main advantage of this converter is the simplicity of design. The topology requires very few components, which reduces the cost and complexity of the development.

In [3] a proportional-integral voltage regulator (PI) is extended by a sensorless predictive control of Buck-Boost converter using a self-correction differential current observer. [4] Presents three adaptive optimal PI controllers that can be used for switching power converters with the unknown load resistance. Robust control used for the control of DC-DC converters in [5, 6]. Presents LQR control of power converters. Similar to this paper, [7] introduces an integral action for DC/DC Boost converter. In [8] a multi-loop based PI control and an LQG method are used to control a DC/DC Buck converter. The nonlinear control is used in [9] for Boost converter. This paper describes the LQR controller for a Buck-Boost with a negative output voltage. Finally, the Kalman filter is a special type of observer that enables optimal filtering of the different types of noise in the measurement and the system if the covariance of these noises is known [10].

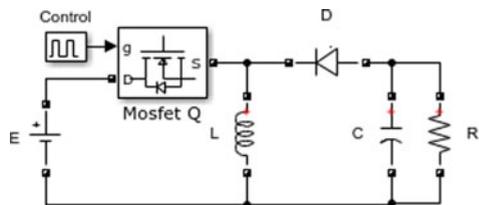
In Sect. 2, the Buck-Boost converter model is used directly for simulation purpose using the LQR controller. The LQR is detailed in Sect. 3. The Kalman filter observer is examined in Sect. 4. The results of the numerical simulation are presented in Sect. 5. Finally, conclusion are presented in Sect. 6.

## 2 The Buck-Boost Model

The Buck-Boost converter with negative output voltages topology is as shown in Fig. 1.

Mosfet Q is ON (Fig. 2), the charging current in the capacitor C is partially discharged. During the second interval in which the Mosfet Q is switched off (Fig. 3), the polarity of the voltage on the inductor is reversed and the diode is conductive [11, 12].

**Fig. 1** Buck-boost converter



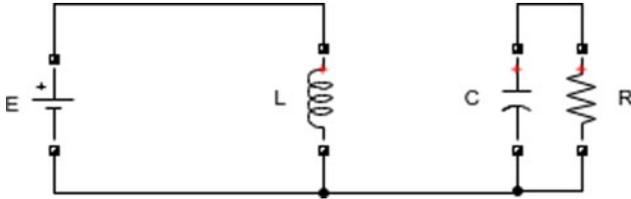


Fig. 2 Switched ON of the Buck-Boost converter



Fig. 3 Switched OFF of Buck-Boost converter

### 2.1 Model of Switched Buck-Boost Converter

By applying Kirchoff’s laws in the two previous circuits, we get the following dynamics:

In the switching ON, the following dynamic is obtained [12]:

$$\begin{cases} L \frac{di}{dt} = E \\ C \frac{dv}{dt} = -\frac{v}{R} \end{cases} \tag{1}$$

In the switching OFF, the following dynamic is obtained [12]:

$$\begin{cases} L \frac{di}{dt} = v \\ C \frac{dv}{dt} = -i - \frac{v}{R} \end{cases} \tag{2}$$

From the two Eqs. (1) and (2), we can obtain a single unified model, which is  $u \in \{0, 1\}$ . The dynamics of the converter are therefore as follows [13]:

$$\begin{cases} L \frac{di}{dt} = (1 - u)v + uE \\ C \frac{dv}{dt} = -(1 - u)i - \frac{v}{R} \end{cases} \tag{3}$$

### 3 LQR Control

#### 3.1 LQR Controller

The Eq. (4), describe the state space model of a system: [14]

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad (4)$$

The vector of optimal control is as follows:

$$u(t) = -Kx(t) \quad (5)$$

To define the optimal control inputs and optimize the state variables, the following cost function  $J$  must be minimized [15]:

$$J = \int_0^{\infty} (x^T Qx + u^T Ru) dt \quad (6)$$

$Q$  and  $R$  two positive definite matrix,  $Q$  and  $R$  are chosen, the  $LQR$  control problem reduces to finding  $K$  that minimizes (6).

Solution  $P$  of Riccati equation [15]:

$$PA + AP + Q - PBR^{-1}B'P = 0 \quad (7)$$

The gain matrix of the optimal control vector can be calculated using the following equation [15]:

$$K = R^{-1}B'P \quad (8)$$

Therefore, optimal control equation becomes [16]:

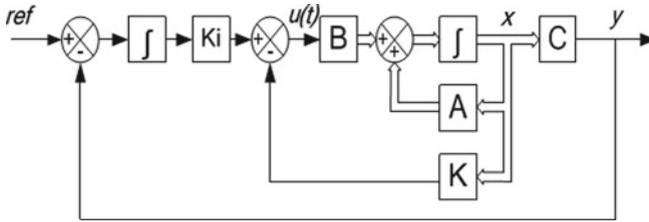
$$u(t) = -Kx(t) = -R^{-1}B^T Px(t) \quad (9)$$

#### 3.2 LQR Control with Integral Action

The schematic diagram  $LQR$  with integral is shown in Fig. 4 [10, 16].

The  $LQR$  with integral action require the following extended matrix configuration:

$$\hat{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \hat{C} = [0 \ 0], \hat{K} = [k \ -k_i]$$



**Fig. 4** Schematic of LQR with integral action

With

$k$ : State-feedback vector gain.

$k_i$ : Integral action.

The cost function is known by the following relationship [15]:

$$J = \int_0^\infty (e^T(t) Q e(t) + u^T(t) R u(t)) dt$$

The vector  $e(t)$  is defined by:

$$e(t) = \begin{bmatrix} \hat{x}(t) - x(\infty) \\ \xi(t) - \xi(\infty) \end{bmatrix} \tag{10}$$

Where  $\xi(t)$  represents the integral action.

The gain  $K$  in (12) is obtained by solving the:

$$\hat{A}^T P + P \hat{A} - P \hat{B} R^{-1} \hat{B}^T P + Q = 0 \tag{11}$$

$$\hat{K} = R^{-1} \hat{B}^T P \tag{12}$$

## 4 Kalmen Filter

### 4.1 Kalman Observer

We consider that only the output voltage is sensed. Then, to estimate the inductor current and reduce the noisy environment effects, a Kalman filter is introduced [10]:

$$\begin{cases} \hat{\dot{x}}(t) = A \hat{x}(t) + B u(t) + \omega(t) \\ \hat{y}(t) = C \hat{x}(t) + v(t) \end{cases} \tag{13}$$

where  $\omega(t)$  is a process noises, and  $v(t)$  is the voltage sensor noise.

The state equations of the Kalman filter can be carried out as follows [17]:

$$\hat{\dot{x}} = (A - K_K C)\hat{x} + Bu + K_K y \quad (14)$$

Where  $K$  is the Kalman gain matrix.

The Kalman filter is usually designed for linear time-varying systems is as follows [17]:

$$d\hat{x} = A(t)\hat{x}(t)dt + B(t)u(t)dt + K_K(t)(y(t)dt - C(t)\hat{x}(t)dt) \quad (15)$$

$$K_K(t) = P_K(t)C^T(t)R_K^{-1}(t) \quad (16)$$

$$\frac{d}{dt}P_K(t) = A(t)P_K(t) + P_K(t)A^T(t) - P_K(t)C(t)^T R_K^{-1}(t)C(t)P_K(t) + Q_K(t) \quad (17)$$

$$\hat{x}(t_0) = x_0, P(t_0) = P_0$$

## 5 Simulation Results

In the MATLAB/SIMULINK using SimPowerSystems toolbox. In simulation, the Kalman filter observer algorithm was implemented using the continuous-time Kalman filter block.

The buck-boost converter with negative output voltages topology was simulated and switched by the PWM technique. The parameters used during the analysis of simulation results are shown in Table 1.

**Table 1** The Buck-Boost parameters

E	Input voltage	20 [V]
L	Inductance	15.91 [mH]
C	Capacitance	470 [ $\mu$ F]
R	Loading resistance	20 [ $\Omega$ ]
Vref	The reference output voltage	-40 [V]
Iref	The reference inductor current	6 [A]
Uref	Initial duty cycle	66.67%
VD	Diode voltage drop	0.7 [V]
RD	Diode on-resistance	0.05 [ $\Omega$ ]
RSW	Mosfet on-resistance	0.1 [ $\Omega$ ]
FSW	Commutation frequency	20 [kHz]

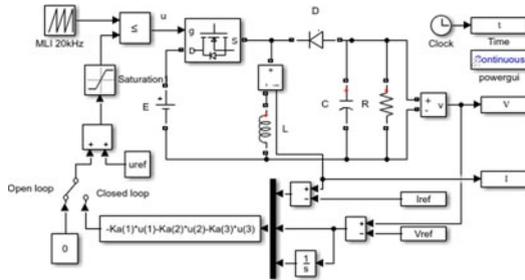


Fig. 5 Open and closed loop control

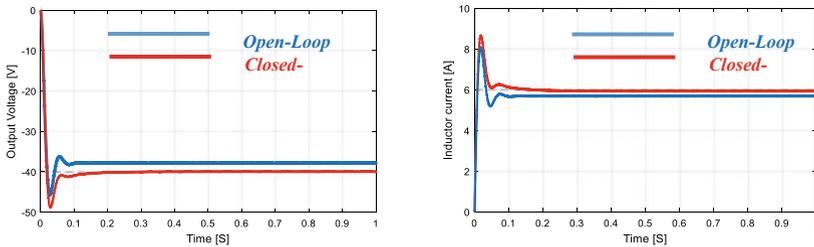


Fig. 6 Simulation results of open and closed-loop with LQR controller

### 5.1 Analysis for Open and Closed-Loop Control

In this case, the simulation of the buck boost converter is carried out for two cases for the control behavior of the converter in the Open and Closed-Loop control using LQR. The SimPowerSystems model is shown in Fig. 5.

According to results of the simulation, the waveform of output voltage and the inductance current have the same waveform characteristic for the open and closed loop control (Fig. 6).

The value of the output voltage in the closed-loop is approximately  $-40\text{ V}$  and about  $-37\text{ V}$  in the open-loop. Then the average value of the inductance current is about  $6\text{ A}$  and  $5.6\text{ A}$  in closed and open-loop respectively. A comparison between the simulation results shows that the control with closed-loop using LQR has better dynamics response compared with open-loop as well as in the maximum peak overshoot/undershoots, settling time.

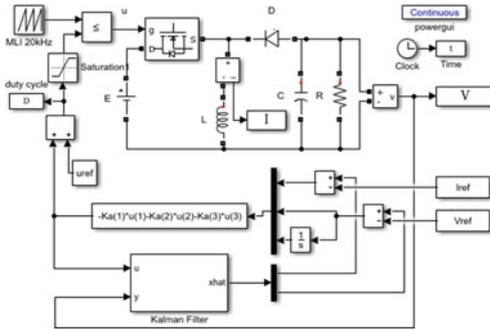


Fig. 7 SimPowerSystems model of buck-boost converter with LQR and Kalman filter observer

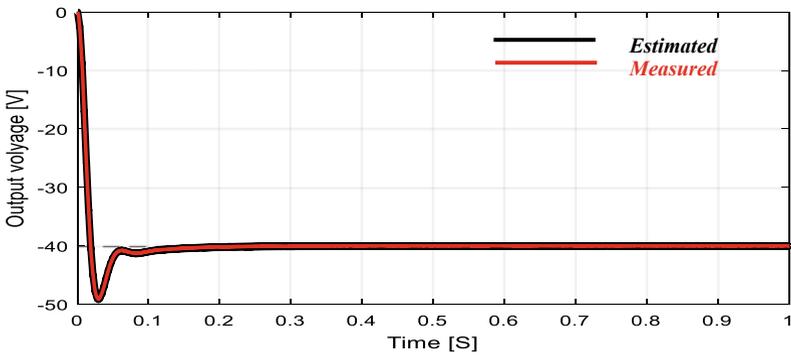


Fig. 8 Simulation results for estimated and measured output voltage

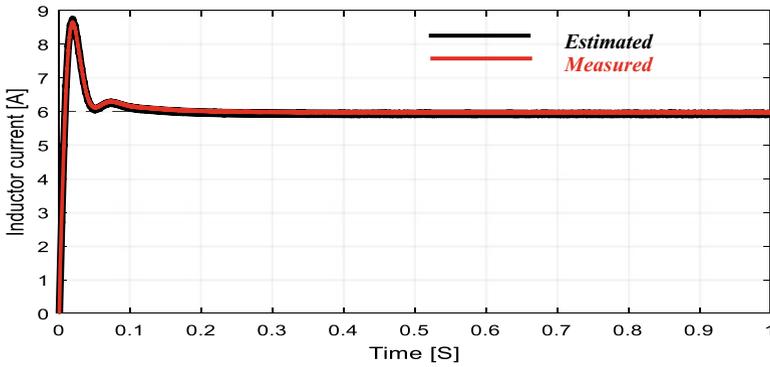


Fig. 9 Simulation results for estimated and measured inductor current

## 5.2 Analysis for Buck-Boost Converter with LQR and Kalman Filter Observer

This section presents a simulation of the dynamic performance of an LQR controller with a Kalman filter observer. The algorithm of the observer was implemented using the Continuous-Time Kalman filter block. The Simulink model with LQR and Kalman filter observer is shown in Fig. 7.

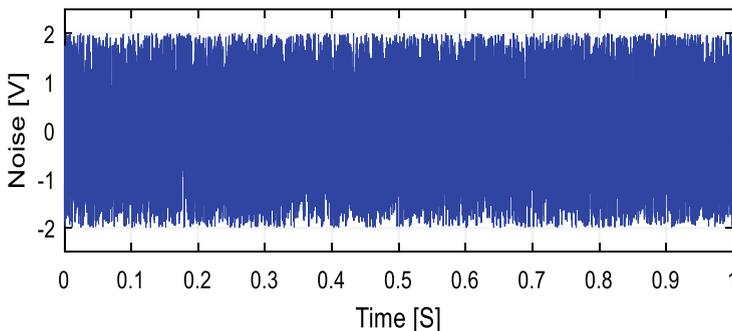
In the Kalman filter observer, the matrices  $Q_K$  and  $R_K$  are difficult to know exactly, since the noises  $w$  and  $v$  are not known. The only possible method is to adjust the  $Q_K$  and  $R_K$  values using simulations. In this simulation, we use the values as follows:

$$Q_K = \begin{bmatrix} 10^9 & 0 \\ 0 & 10^4 \end{bmatrix} \quad R_K = [10^3]$$

The estimated and measured output voltage and inductance current at an output voltage reference of  $-40$  V are shown in Fig. 8 and Fig. 9. In both static and dynamic cases, it can be seen that the signals estimated by the KF observer track the real signals very well.

## 5.3 Effect of a Measurement Noise

In many applications, the measurement noise caused by the hardware or the environment has a significant impact on the system. The estimation accuracy of KF is tested in this paper under noisy output voltage measurement. Figure 10 shows the noise injected at the output voltage in the range of  $[0-2$  A]. The noise is zero mean, white Gaussian. The aim of the voltage injection is to observe the low pass filter characteristics of KF. Figure 11 shows the output voltage, the inductor current shown in Fig. 12. The accuracy of the state estimate can be increased by increasing the covariance of



**Fig. 10** Injected noise to output voltage

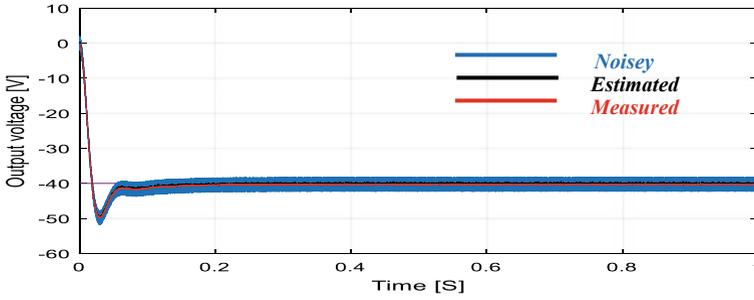


Fig. 11 Measured and estimated, output voltage with measured noisy

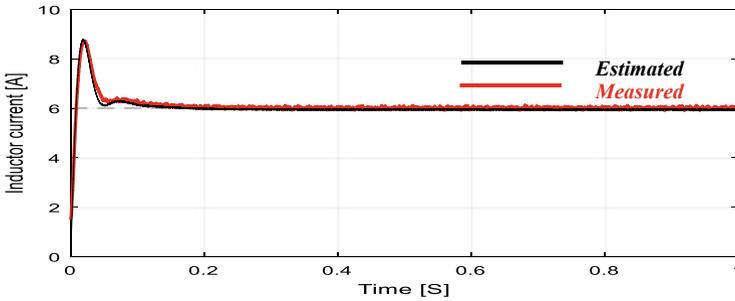


Fig. 12 Measured and estimated inductor current

the measurement noise under noisy conditions, which makes the system model will have more importance.

## 6 Conclusion

This paper has demonstrated the process of applying the state feedback control of a buck-boost converter using LQR with integral action controller. The clear advantages of state feedback, that has a positive effect on response settling time, reducing the undesirable peak overshoots and serve having a less oscillated performance, referring that this approach doesn't provide a zero static error, this latter has been solved by adding an integral action to state feedback control. The simulation results found that adding a Kalman filter reduces the cost and impact of the sensor on LQR tracking performance.

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