Analysis of HNLS Solitons with Quintic Nonlinearity Using Semi Implicit Operator Splitting Padé Method

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Abstract. In this communication, we use the semi-implicit finite difference operator splitting Padé method to solve the higher-order nonlinear Schrödinger equation with higher order linear and nonlinear effects such as the third order dispersion (TOD), Kerr dispersion, stimulated Raman scattering and the quintic nonlinearity. The role of quintic nonlinearity on the propagation characteristics of optical solitons is investigated for standard fibers. This method is readily generalized for nonlinear management fibers it's found that the quintic nonlinearity has no significant role on the propagation of single solitons.

Keywords: Optical solitons, Padé method, quintic nonlinearity. HNLS equation

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INTRODUCTION

Propagation of solitons in optical fibers has attracted a great attention of many authors [1-3]. The balance between nonlinear Kerr effect and chromatic dispersion is the clue of the stability of optical NLS solitons [2]. Using high power pulses increases the signal to noise ratio making the distance between the repeaters longer which helps to minimizing the coast of the system [2]. At the same time the transmission of very high bit rate requires optical pulses of very low temporal width. However optical fibers present higher order nonlinear and dispersion effects that can alter the stability of very short high power solitons. Thus the propagation of ultra short and ultra intense optical solitons in optical fibers is not so obvious and requires efficient and fast numerical methods in order to be investigated. At high power the higher order non linearity may alter the propagation of the solution especially with the interplay of higher order dispersion effects such as third ordered dispersion. Quintic nonlinearity is the most important non linear effect due to the saturation of optical field and must be taken into account in the study of ultra intense optical solitons. We use the semi-implicit finite difference operator splitting Padé method OSPD [4] to solve the higher-order nonlinear Schrödinger HNLS equation with quintic nonlinearity. The OSPD method shows many advantages over classical methods such as split step Fourier method or finite difference rank Nicolson method. SSFM is time consuming especially for wavelength division multiplexing WDM systems. OSPD method is more efficient compared to other finite deference methods such as Crank Nicolson CN methods. It is more rapid and well adapted for higher order time derivatives such as third order dispersion or Kerr dispersion

THEORY

Starting from the higher-order nonlinear Schrödinger HNLS equation with quintic nonlinearity:

$$\frac{\partial A}{\partial z} = -\frac{i}{2}\beta_2\frac{\partial^2 A}{\partial T^2} + \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial T^3} - \frac{\alpha}{2}A + i\gamma\left[|A|^2 A + \gamma'|A|^4 A + \frac{i}{\omega_0}\frac{\partial}{\partial T}(|A|^2 A) - T_R A\frac{\partial |A|^2}{\partial T}\right] \quad (1)$$

Where A is $A(z,T)$ that represents the slowly varying envelope of the optical pulse. $T = t - z/v_g$ the temporal coordinate in retarded frame that moves at the group velocity v_g of the pulse and z the spatial coordinate representing the distance of transmission. β_2, β_3 and α represent respectively, the group

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velocity parameter, the third order dispersion parameter and the attenuation factor. For silica fibers $\beta_2 \sim 50ps^2/km$,at $\sim 1.55\ \mu m$. $\gamma = \frac{n_2\omega_0}{cA_{eff}}$, and γ' are respectively the nonlinear Kerr effect coefficient and the quintic nonlinearity coefficient. n_2 is the nonlinear index coefficient, for silica fibers $n_2 = 2.2 \times 10^{-20}m^2/W$ at the same wavelength. The term proportional to $\frac{1}{\omega_0}$ governs the Kerr dispersion that is responsible of self-steepening and shock formation. The last term is responsible of self-frequency shift induced by intrapulse Raman scattering and T_R is related to the Raman response function. A wave lengths' $\sim 1.55\ \mu m\ T_R \approx 3fs$.

NUMERICAL SIMULATIONS

We proceed to the normalization of the equation (1) in the following way:

$\tau = \frac{T}{T_0}$;$z = \frac{z}{L_D}$, $L_D = \frac{T_0^2}{|\beta_2|}$,$L_{NL} = \frac{1}{\gamma P_0}$, $L'_D = \frac{T_0^3}{|\beta_3|}$, $s = \frac{1}{\omega_0 T_0}$;and $\tau_R = \frac{T_R}{T_0}$

Where T_0 is the initial pulse width, L_D ,L'_D, and L_{NL} are three length scales. The parameters s and τ_R govern the effects of self-steeping and intrapulse Raman scattering respectively. Both of these effects are quite small for picoseconds pulses but must be considered for ultra short pulses with $T_0 < 0.1ps$ [1]. Searching for the solution in the form:

$$A(Z,\tau) = \sqrt{P_0}U(Z,\tau).\exp\left(-{\alpha Z}/{2}\right)$$

The equations (1) after normalization take the following form:

$$\frac{\partial U}{\partial Z} = i\left(\alpha_1\frac{\partial^2 U}{\partial \tau^2} + \alpha_2|U|^2U\right) + \epsilon\left[\alpha_3\frac{\partial^3 U}{\partial \tau^3} + \alpha_4\frac{\partial}{\partial \tau}(|U|^2U) + \alpha_5 U\frac{\partial |U|^2}{\partial \tau} + \alpha_6|U|^4U\right] - \frac{\alpha}{2}U.\exp\left(-{\alpha Z}/{2}\right) \quad (2)$$

Where:

$\alpha_1 = -\frac{sgn(\beta_2)}{2}$, $\alpha_2 = \exp\left(-{\alpha Z}/{2}\right)\frac{L_D}{L_{NL}}$, $\alpha_3 = \text{sgn}(\beta_3).\frac{L_D}{6\,L'_D}$, $\alpha_4 = -\text{s}\,\frac{L_D}{L_{NL}}$,

$\alpha_5 = -\text{i}\exp\left(-{\alpha Z}/{2}\right).\frac{L_D}{L_{NL}}\tau_R$

$\alpha_6 = \text{i}\,\frac{L_D}{L_{NL}}\gamma'$. And $\epsilon = \exp\left(-{\alpha Z}/{2}\right)$

In our simulation, we suppose the parameter ($\alpha = 0$) fiber with lossless. The equation (2) can be rewritten:

$$\frac{\partial U}{\partial Z} = i\left(\alpha_1\frac{\partial^2 U}{\partial \tau^2} + \alpha_2|U|^2U\right) + \epsilon\left[\alpha_3\frac{\partial^3 U}{\partial \tau^3} + \alpha_4\frac{\partial}{\partial \tau}(|U|^2U) + \alpha_5 U\frac{\partial |U|^2}{\partial \tau} + \alpha_6|U|^4U\right] \quad (3)$$

This equation (3) can be represented also by:

$$\frac{\partial U}{\partial Z} = \left(\hat{L} + \hat{N}\right)U \quad (4)$$

Where: $\hat{L} = i\alpha_1\frac{\partial^2}{\partial \tau^2} + \epsilon\alpha_3\frac{\partial^3}{\partial \tau^3}$ is the linear operator

and: $\hat{N} = i\alpha_2|U|^2 + \epsilon\left[\alpha_4|U|^2\frac{\partial}{\partial \tau} + (\alpha_4 + \alpha_5)\frac{\partial |U|^2}{\partial \tau} + \alpha_6|U|^4\right]$ is the nonlinear operator

In this method we approximate the exact solution of equation (4) by solving the purely linear equation and purely nonlinear equation separately, in which the solution of one sub problem is employed as an initial condition for the next sub problem [4]. Hence the second member in the equation is split into two parts. The first linear term is solved using the semi-implicit finite difference operator splitting Padé algorithm that is unconditionally stable. The second nonlinear part is solved using a fourth order Runge-Kutta scheme that satisfies the CFL condition.

"Semi implicit operator splitting Padé method-OSPD-" shows many advantages compared to other methods for solving the HNLS equation. It is less time consuming, it takes less memory and it is well adapted for higher order time derivatives.

RESULT AND DISCUSSION

We have test ted the OSPD method for two cases. The first case corresponds to the propagation of a single first order soliton along the optical fiber. The second case is the case of the interaction between two neighboring solitons in the same channel. In each case we study the influence of the quintic nonlinearity and compare the CPU time between OSPD and CN methods.

Single Soliton Propagation

First we consider the case of a single soliton propagating in the optical fiber in the presence of the group velocity dispersion GVD, self phase modulation (SPM) and the quintic nonlinearity. Figure 1 shows the propagation af the first order soliton along 100km of fiber without distortion. This is in conformity with the prediction of the variational method [5]. However, this not the case when the other higher nonlinear effects account (such as Kerr dispersion Raman scattering [7] and the third order dispersion) are taken into account at the same time with the quintic nonlinearity. The interplay between this higher order effects and the quintic nonlinearity contributes to decrease the amplitude accompanied by an increase of the width of the pulse..

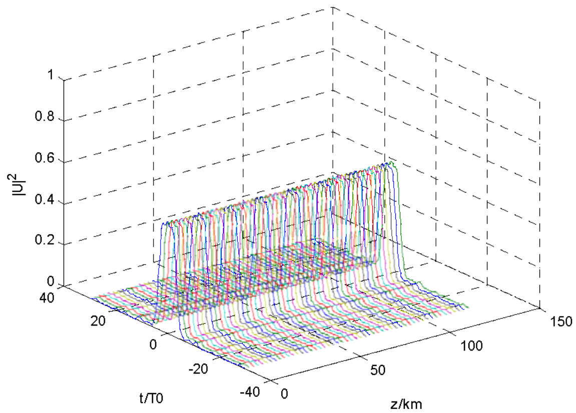

FIGURE 1. Evolution of the optical soliton along 100km of fiber

Interaction Between Solitons

Now we study the influence of the quintic nonlinearity on the collision length of two neighboring solitons situated in the same channel.

Figure 2 shows a contour plot of the propagation of two neighboring solitons for the following parameters fiber $\alpha_1 = 1/2$,$\alpha_2 = 2$,$\alpha_3 = 1$, $\alpha_4 = 6$, and $\alpha_5 = -3$. The quintic nonlinearity parameter is set to $\alpha 6 = -0.05$.

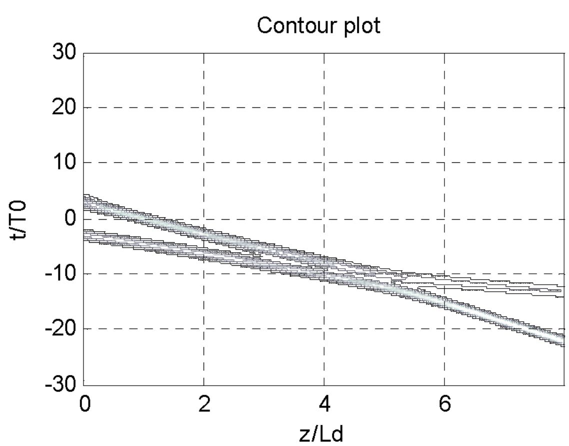

FIGURE 2 Contour plot showing the collision between two solitons in the same channel.

Although the effects of the quintic nonlinearity on the propagation of single solitons is only marginal, in the case of many solitons propagating in the same channel the quintic nonlinearity has a significant effect o the collision length of solitons.

As shown in Figure 3 it has been found that in the case where all the higher order effects are included, the collision length increase with the quintic nonlinearity

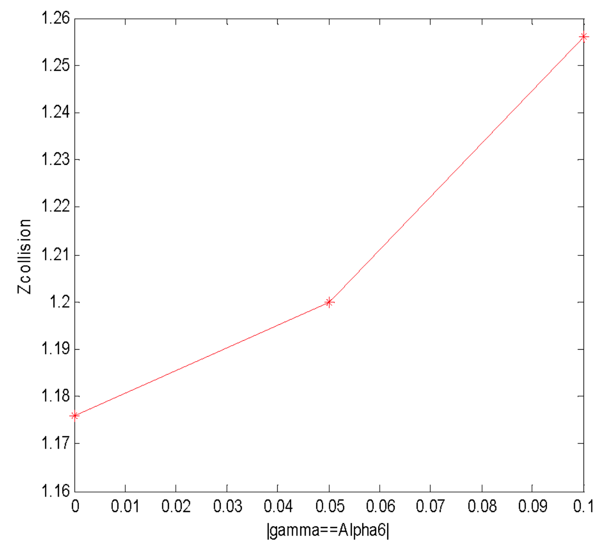

FIGURE 3 Influence of the quintic nonlinearity on the collision length between neighboring solitons in the same channel.

In Table (1) we collect some values of the ratio of the CPU time between the two methods OSPD and CN for two values of the quintic nonlinearity parameter and for different values of the space step Δz

We note that for each value of quintic parameter the ratio of CPU time decrease when we refine the space grid, this result shows that the "OSPD" method is faster than Crank Nicolson method.

In this work we have investigated the propagation of solitons in standard fibers. In a future work we shall investigate the case of dispersion [6] and nonlinear managed [8] and fibers.

Quintic parameter	$\Delta t, \Delta z = 0.031, 1.10^{-3}$	$\Delta t, \Delta z = 0.031, 4.10^{-5}$
$\alpha_6 = -0.10$	$OSPD/CN = 0.82$	$OSPD/CN = 0.23$
$\alpha_6 = -0.05$	$OSPD/CN = 0.84$	$OSPD/CN = 0.60$

CONCLUSION

The semi-implicit finite difference operator splitting Padé method has been used to investigate the propagation of ultra intense and ultra short optical solitons under the effect of the quintic nonlinearity. It has been shown that the OSPD presents many advantages over the split sep Fourier method and the Crank Nicolson method as it is more efficient, rapid and takes less memory space. It has been confirmed that the role of quintic nonlinearity on the propagation characteristics of optical single solitons is marginal. However it influences the propagation of neighboring solitons in h same channel

This method is readily generalized for nonlinear management fibers.

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