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Numerical approximation and umbral calculus

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Abstract: We show that any operator P, invariant by translation, can be developed as P=? n?0 a n Q n ,(1) where Q n is a sequence of well defined operators. By using the umbral calculus, this generalizes the result of G. C. Rota et al. [Finite operator calculus, J. Math. Anal. Appl. 42, 685–760 (1973)]. Our idea consists in choosing a ?-sequence (Q n) n?? defined as Q 0 =I(Identity),forn=0,Q n =1 h n1 h n2 ?h nn ? h n1 ? h n2 ?? h nn ,forn?0, where ? h is the difference operator: 1 h? h f(x)=f(x+h)-f(x) h,forh?0,f '(x),forh=0, while (h ij) is a double sequence of arbitrary parameters. Our goal is to find parameters (h ij) minimizing the absolute values of the functions a n (h n1 ,h n2 ,?,h nn) and to determine, experimentally, on a representative class of functions: • What are the values of the h ij that minimize an for n=4 or n=5? • Does, by the accelerated convergence of the series of operators (1) with an optimum choice of the h ij , this imply an accelerated convergence of the corresponding series of functions (in case of convergence of course)? A positive answer would open a perspective in the process of the acceleration of the convergence of numerical series. It is noted that (1) is independent of the choice of these parameters, therefore we exploit this idea in the approximation of certain operators encountered in numerical analysis.

Keywords : Numerical approximation