

# A New Approach to Solve Inverse Kinematics of a Planar Flexible Continuum Robot

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**Abstract**— Research on the modeling of continuum robots, focused on ways to constrain the geometric models, while maintaining maximum specificities and mechanical properties of the robot. In this paper we propose a new numerical solution for solving the inverse geometric model of a planar flexible continuum robot, we assuming that each section is curved in an arc of a circle, while having the central axis of the inextensible structure. The inverse geometric model for one section is calculated geometrically, whereas the extreme points, of each section, used in calculating the inverse geometric model for multi-section is calculated numerically using a particle swarm optimization (PSO) method. Simulation examples of this method are carried to validate the proposed approach.

**Keywords:** flexible continuum robot; planar robot; Inverse kinematics; particle swarm optimization.

## I. INTRODUCTION

Modeling continuum robots requires having a continuous model of the central axis of the robot. Traditional approaches to modeling, in which the marks are associated with each joint, are inappropriate for the case of continuum manipulators, precisely because of the absence of discrete bonds in their architecture. A natural approach is to use a theoretical model curve to the central axis of robot hyper-redundant. The fundamental question, concerning the modeling of the robots continuum, also lies in the infinity of the number of degrees of freedom which the geometrical models in their continuous form require. However, the hyper-redundant robotic systems, discrete lines or continuum cannot be control by considering a finite number of degrees of freedom, thus admitting a reduced set of physical solutions. Among contributions on the geometric model of these structures, we can cite the model of [1, 2]. Used the Frenet-Serret formulas and modeling the central axis of the robot by a kinematic chain consisting of several rigid bodies. [3, 4] they assumed that each section of a manipulator bends in an arc of a circle of constant curvature and that the central axis of the structure is inextensible. The latter is based on modified Denavit-Hartenberg Convention. The validation of model has been done on a prototype robot called the 'elephant trunk'. Similarly [5] uses this assumption to study the kinematics of a snake robot. Furthermore, the inverse geometric model for the case of a multi-section is studied by [6], where the end points of each section are assumed to be known. The authors did not validate experimentally the model but they simulated the overall behavior using appropriate software. [7] Deals with the validation of the geometric model of a manipulator multi-section placed on a mobile robot as well as the inverse geometric model for one bending section and that experimentally validation. [8] Has done enhancements on the inverse geometric model described by [3] for one bending section, the validation of this model has done on a micro-robot and showed that the model was not robust against uncertainties of environment and inaccuracies of materials. This inverse geometric model is formulated as an optimization problem with a cost function and constraints using the principle of interval analysis. Finally, a report a state of the art in modeling these classes of robots is presented by [9]. This paper presents a geometric approach to determining the inverse kinematics for one section and multi-section continuum robots. In section II.A determine a solution to the inverse kinematics problem for one bending section. Section II.B discusses extending the results from section II.A to a planar multi-section robot, assuming knowledge of the end-point locations for each section of the robot. Section III presents a procedure to compute the operational coordinates of the origins of the intermediate platforms of each section, by using a method of optimization PSO. Finally, Simulation examples using this method are carried out to validate the proposed approach.

## II. INVERSE GEOMETRIC MODEL

### A. Inverse geometric model for one section

The objective of the inverse geometric model is to calculate the cables parameters (lengths) as function of  $(x, y)$ , Cartesian coordinates of the origin of the platform mobile, defined in the origin frame of the base. This model is validated only for one bending section; it is not extensible to multi-section based on the same calculation. one bending section of a planar continuum robot is modeled as an arc of circle with one end point  $O$  fixed to the origin of the reference frame, the other end point  $P$  located at anywhere in the workspace and the center of arc in the  $y$  axis. This section of continuum robot is parameterized by its arc length  $s$ , its curvature  $\kappa$ , and its orientation  $\theta$  as shown in figure.1. These parameters can be calculated geometrically by:

$$\theta = \text{atan2}(y, x) \quad (1)$$

$$\kappa = \sqrt{\frac{2(1-\cos(2\theta))}{x^2+y^2}} \quad (2)$$

$$s = \frac{2\theta}{\kappa} \quad (3)$$

### B. Inverse geometric model for multi-section

The inverse geometric model presented in the previous section can be iteratively applied to several sections connecting in series to model a continuum robot of  $n$  sections, knowing the extreme points of each section. The schema of algorithm for computing the inverse geometric model of the overall structure is presented in figure.2. Where  $\theta_i$  angle between the lines which passes by both ends of section  $i$  and the tangent of the final point of the section  $i-1$ .

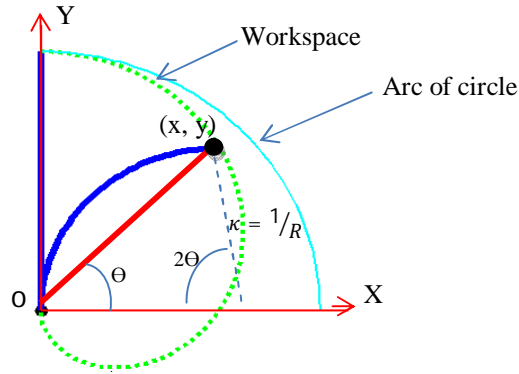


FIGURE 1: Geometrical parameters

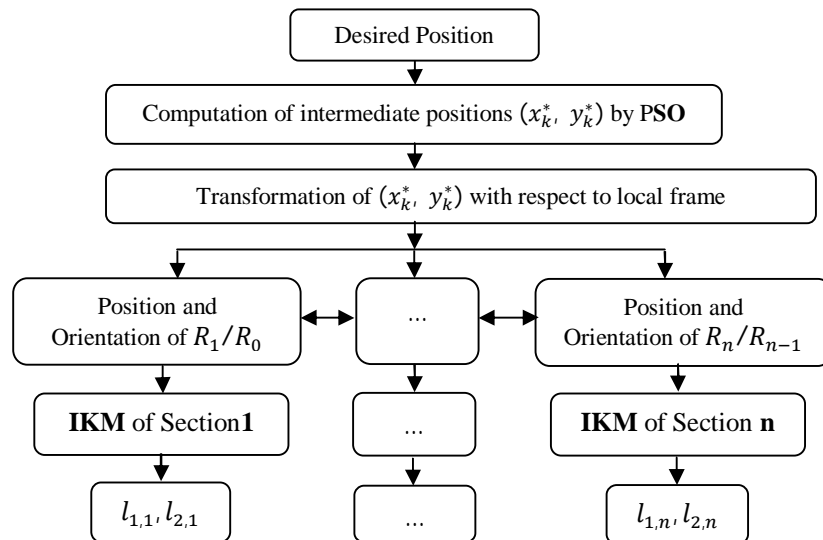


FIGURE 2: Algorithm for computing the inverse geometric model of planar continuum multi-sections robots.

### III. END-POINT LOCATIONS OF EACH SECTION FOR A PLANAR MULTI-SECTION CONTINUUM ROBOT

The operational coordinates of the origins of the intermediate platforms  $(x_i, y_i)$  of each sections, are calculated numerically by a method of optimization PSO, using constraints on the conservation length, such that:

$$l_i = \sqrt{((x_i - x_{i-1})^2 + (y_i - y_{i-1})^2)} \theta_i / \sin(\theta_i) \quad (4)$$

$$\theta_i = \text{atan2}((y_i - y_{i-1}), (x_i - x_{i-1})) \quad (5)$$

For  $i = 1 \dots n$ . Where  $(x_i, y_i)$  represent the Cartesian coordinates of the origin of reference frame  $R_i$  in the Frame  $R_{i-1}$ .

#### A. PSO

This method is inspired by the social behavior of some biological organisms, especially the group's ability of some animal species to locate a desirable position in the given area. It was proposed first by [10]. Thus, using the simple rules of displacement (in the space of the solutions), the particles can progressively converge to a local minimum. For each iteration  $t$ , the velocity changes by applying equation (15) to each particle.

$$v_i(t+1) = \omega v_i(t) + c_1 \varphi_1 (P_{ibest} - x_i) + c_2 \varphi_2 (P_{gbest} - x_i) \quad (6)$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (7)$$

Where  $\omega$  is called the inertia weight,  $c_1$  and  $c_2$  are weighting factors and  $\varphi_1, \varphi_2$  are random variables uniformly distributed within interval  $[0, 1]$ .  $P_{ibest}$  and  $P_{gbest}$  represent the best position visited by a particle and the best position visited by the swarm till the current iteration  $t$  respectively. The position update is applied by equation (7) based on the new velocity and the current position. The calculation steps are given by the algorithm shown in figure. 3.

#### B. Numerical application

Knowing the general movement of platform  $n$  produced by the operational coordinates  $(x_n, y_n)$  we determine the Cartesian coordinates of the intermediate platforms, by using the method of optimization PSO.

The cost function is given by:

$$F(l_i, l_{r,i}) = \sum_{i=1}^n f_i(l_i, l_{r,i}) \quad (8)$$

With: 
$$f_i(l_i, l_{r,i}) = \frac{1}{2} (l_i - l_{r,i})^2 \quad (9)$$

Where  $n$  is the number of bending sections,  $l_i$  is given by equation (4) and  $l_{r,i}$  is the real length of the bending section  $i$ .  $x_i$  and  $y_i$  are defined by solving the following optimization problem:

$$\min_{x_i, y_i} F(l_i, l_{r,i}) \quad (10)$$

To illustrate this inverse geometric modeling, we applied on a planar robot consists of two bending sections for the following settings of the trajectory in operational space:

$$\begin{cases} x_2 = 350 \cos(t) \\ y_2 = 150 \sin(t) \end{cases}; \text{ Where } t \text{ is time variable varies from } 0 \text{ to } \pi/2.$$

The simulations were done on a PC with a processor i3 3.30 GHz. The trajectory has been accomplished in 1.60 sec of time duration, corresponding to 7 position captures. The execution time of the optimization process for each point found is shown in figure.5. Average time is equal to 15.9841 msec. Figure 6 shows the convergence of cost function. We can notice that starting from the 57 iteration onwards, cost function goes below 0.09. In this simulation, the tolerance error is fixed to  $10^{-6}$ .

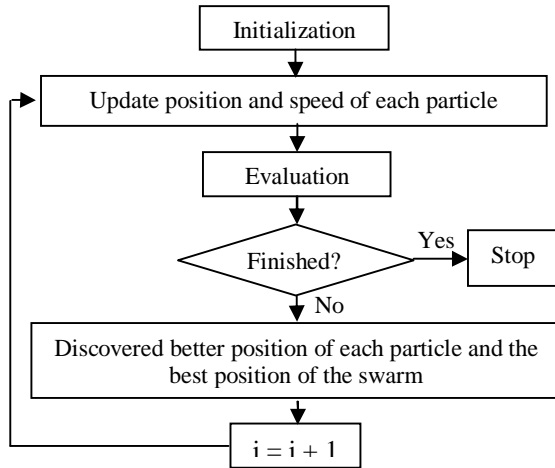


FIGURE 3: PSO algorithm.

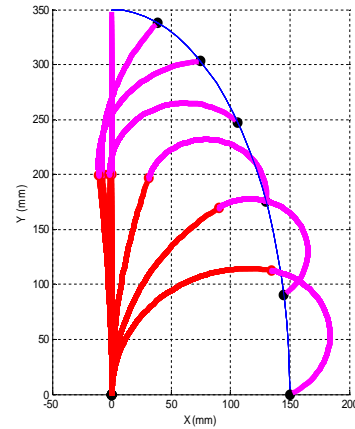


FIGURE 4: The virtual axis of the robot

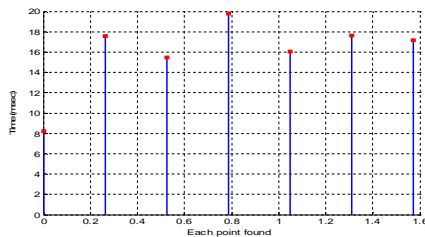


FIGURE 5: Execution time of the algorithm

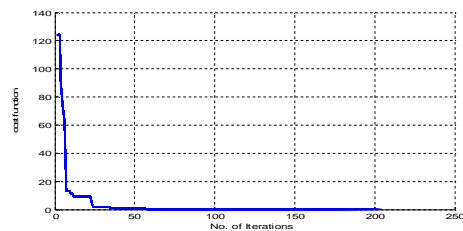


FIGURE 6: Convergence of the cost function

#### IV. CONCLUSION AND FUTURE WORKS

In this paper, a new approach to solving the inverse geometric model of planar continuum robots is proposed. This approach allows us to calculate the Cartesian coordinates of the origin of intermediate platforms using the PSO optimization method, where the search space that is chosen arbitrarily. Future work is to improve the execution time, one taking the space of research in the workspace of each section, in order to find a solution close to real time. And to have the uniqueness of the solution we work to introduce other constraints on this method and the generalization of the approach in the three-dimensional.

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