Desired Closed-Loop Based Self-Tuning Fractional PI$^D$D$^H$ Controller for Wind Turbine Speed Control

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Abstract—Lately, fractional PI$^D$D$^H$ controller has gained more interests because it involves two more tuning parameters, the fractional order integration $\lambda$ and the fractional order differentiation $\mu$, in their structure. This type of controller has been introduced in the control system theory in a continuous effort to enhance the system control quality performances. In this paper a self-tuning fractional PI$^D$D$^H$ controller for wind turbine speed control is proposed. The fractional PI$^D$D$^H$ controller’s five parameters are self-tuned as the system dynamics or operating points change using formulas based on a desired closed loop response. A simulation study of the wind turbine speed control has been done to demonstrate and validate the performance and effectiveness of the proposed control scheme.

I. INTRODUCTION

Design and tuning of fractional PI$^D$D$^H$ controllers have been the focus of many control theory researchers since it has been proposed for the system control quality enhancement and improvement [1]. Various tuning methods of the fractional PI$^D$D$^H$ controllers have been proposed to fulfill different control design specifications [2-8]. Hence, with their growing number of applications and widespread usage, an effort has to be made to look for and develop alternative and better design techniques for this controller to improve the overall feedback control system performances. In this paper a self-tuning fractional PI$^D$D$^H$ controller is proposed as an extension of a design method in [8] where the five parameters of the fractional PI$^D$D$^H$ controller are tuned such that the behaviour of closed loop system is equivalent to a desired fractional order model whose open loop transfer function is Bode’s ideal function called in this context the fractional order oscillatory or relaxation system [9]. This type of fractional order system is widely used in the fractional order control domain because it has the iso-damping property which is a very important robustness feature [10-14]. The basic ideas of the fractional PI$^D$D$^H$ controller’s parameters self-tuning method as the system dynamics or operating points change using analytical formulas obtained in [8] are presented. A simulation study of the wind turbine speed control has been done to demonstrate and validate the performance and effectiveness of the proposed control scheme. In this case, the analytical formulas used in the self-tuning of the fractional PI$^D$D$^H$ controller’s parameters are derived using the linear model of the wind turbine system given in [15]. In the literature several methods used for wind turbine speed control, among which was the PID controller [19, 20], sliding mode control [21], fuzzy and LQR control [22]…, all of these methods get a satisfactory result. On the other hand the fractional control is very used in robust control because of its additional flexibility [2], [5], [6], [10]. Many works has justified that Fractional PID controller which is an extension of classical PID can satisfies more specification than classical control [1]. Recently, a new work was proposed [15] where the authors use the fractional controller for wind turbine speed control in fixed speed operation, the fractional controller get the required specification with an important proprieties which is robustness in term of gain variation. for this reason in our work we chose to use this type of controller to improve the system performance.

Besides, for the computational efficiency of the proposed control method the implementation of the fractional PI$^D$D$^H$ controller has been done through the adjustable fractional order differentiator and integrator technique presented in [16].

II. MODEL OF WIND TURBINE

A. Non linear model

From [15], [17] and [18], the nonlinear model of the wind turbine is given by the following expressions:

$$T_{ar} = GM + \frac{P_e}{\Omega} = GP/G_{\text{mec}} = \frac{1}{2} C_{p} (\lambda, \beta) \rho A_{\text{g}} v_{\text{w}}^{3}/\Omega,$$

$$C_{p} (\lambda, \beta) = 0.5176 \left( \frac{116}{\lambda} - 0.4 \beta - 5 \right) \frac{21}{\lambda + 0.0068 \lambda},$$

$$\frac{1}{\lambda_{1}} = \frac{1}{\lambda + 0.08 \beta} + 0.035 \beta^{3} + 1,$$

$$\lambda = \frac{\Omega_{i}}{\Omega_{I}},$$

where $P_e$ is the aerodynamic power obtained from the wind, $C_{p}$ is the power coefficient, $\rho$ is the density of air; $A_{g}$ is the surface covered by the wind wheel $v_{w}$ is the wind speed; $\lambda$ is the tip speed ratio (TSR), $\beta$ is the pitch angle, $\Omega_{I}$ is the wind turbine mechanical angular velocity; $T_{ar}$ is the wind turbine output torque; $T_{g}$ is the driving torque of the generator, $G$ is the gear ratio and $\Omega_{m}$ is the generator mechanical angular velocity. The relation between the to
The fundamental dynamics of the wind turbine are then given by the following first order differential equation [15]:

\[ T_e - T_{SEG} = J \frac{d\Omega}{dt} \]  

where \( T_{SEG} \) is electromagnetic torque of Self-Excited Induction Generator (SEIG), in this work \( T_{SEG} \) is considered as perturbation and obtained from the mathematical model of Generator presented in [15].

### A. Model linearization

In order to use a linear controller it is necessary to linearize the model about a specified operating point. The linearization of equation (3) is given by [17,18]:

\[ \beta \Delta \delta + \upsilon \Delta \xi + \Delta \Omega \gamma = \Delta \Omega \omega \]  

The linearization coefficients are given by:

\[
\begin{align*}
\xi &= \frac{\partial \Delta \Omega}{\partial \upsilon} = \frac{\partial}{\partial \upsilon} \left( J \Omega \right) |_{op} = 0.5 \alpha \Lambda | \Omega |^{-1} \frac{\partial C_p(\gamma, \beta)}{\partial \upsilon} |_{op} \\
\delta &= \frac{\partial \Delta \delta}{\partial \upsilon} = \frac{\partial}{\partial \upsilon} \left( J \Omega \right) |_{op} = 0.5 \alpha \Lambda | \Omega |^{-1} \frac{\partial C_p(\gamma, \beta)}{\partial \upsilon} |_{op} \\
\end{align*}
\]

where \( \beta_{op} = R \frac{\Omega_{op}}{\upsilon_{op}} \), \( \Delta \Omega, \Delta \upsilon_{op} \), and \( \Delta \beta \) represent deviations from the chosen operating point, \( \Omega_{op}, \upsilon_{op} \), and \( \beta_{op} \). By application of Laplace transformation to equation (4) we obtain:

\[ J \frac{d}{dt} \Delta \Omega = \gamma \Delta \Omega + \xi \Delta \upsilon_{op} + \delta \Delta \beta(s) \]  

For \( D = \frac{\gamma}{J} \), the turbine speed can be represented as:

\[ \Delta \Omega = \left[ \frac{\xi}{J} \Delta \upsilon_{op}(s) + \frac{\delta}{J} \Delta \beta(s) \right] \frac{1}{s - D} \]  

Hence, the linear model is represented by the block diagram shown in Fig. 1.

**Figure 1** wind turbine linear model [15].

The factor \( \Delta \upsilon_{op}(s)^{*}(\xi/JJ) \) is considered as a constant perturbation and the parameters \( T = (-1/D), K = (-\delta/JD) \) are the time constant and static gain respectively. When the turbine operating point deviate the static gain and time constant parameter gets significant variation; so the closed loop system change its behavior, for this reason we must use a robust control strategy for this application.

### III. CONTROL DESIGN

#### B. Principle of the control strategy

Figure 2 shows the self-tuning control scheme where \( G_{p}(s) \) is the plant’s transfer function and \( C(s) \) is the controller’s transfer function. The controller \( C(s) \) is a fractional order \( \mathbf{P}^\mathbf{D} \) controller whose transfer function is given as follows:

\[ C(s) = K_s + \frac{T_1}{s} + T_D s^m \]  

**Figure 2**: Self-tuning control scheme

First, the fractional \( \mathbf{P}^\mathbf{D} \) controller’s five parameters are derived analytically off line as functions of the plant’s parameters and the desired performances using the procedure given in [8]. Then, as the system dynamics or operating points change the fractional \( \mathbf{P}^\mathbf{D} \) controller’s parameters are self-tuned using the already calculated formulas to keep the closed loop performances as the desired ones. In this proposed control strategy, any system dynamic or operating point change is detected through an on line plant model’s parameters estimation.

#### C. Controller design:

The principal of this feedback control system is to tune the parameters of the fractional order \( \mathbf{P}^\mathbf{D} \) controller of equation (8) to guarantee that the closed loop transfer function of a unity feedback control system behaves in a given frequency band as a desired reference model whose transfer function is the Bode’s ideal transfer function given as:

\[ G_d(s) = \frac{1}{1 + \left( \frac{s}{\omega_n} \right)^m} \]  

where \( m \) is a real number such that \( 0 < m < 2 \) and \( \omega_n \) is a positive real number. This function is considered as the closed loop transfer function of the unity feedback control of figure (2) whose open loop transfer function is \( \left( s / \omega_n \right)^m \).

The two parameters \( m \) and \( \omega_n \) are chosen such that the above desired \( G_d(s) \) system meets the dynamic performance requirements of the projected feedback control system. If
Step 1: Calculate the variables $\theta_i$, for $0 \leq i \leq 4$

$$\theta_0 = \frac{1}{2}$$
$$\theta_1 = -\frac{m}{4\omega_u}$$
$$\theta_2 = \frac{m}{4\omega_u^2}$$
$$\theta_3 = \frac{m(m^2 - 4)}{8\omega_u^3}$$
$$\theta_4 = -\frac{3m(m^2 - 2)}{4\omega_u^4}$$

Step 2: Calculate the variables $S_i$, for $0 \leq i \leq 4$

$$S_0 = \frac{G_p(\omega_u)}{\omega_u}$$
$$S_1 = \left(\frac{G_1(s)}{s}\right)^{(1)}_{\theta = \omega_u}$$
$$S_2 = \left(\frac{G_2(s)}{s}\right)^{(2)}_{\theta = \omega_u}$$
$$S_3 = \left(\frac{G_3(s)}{s}\right)^{(3)}_{\theta = \omega_u}$$
$$S_4 = \left(\frac{G_4(s)}{s}\right)^{(4)}_{\theta = \omega_u}$$

Step 3: Calculate the terms $G_o(i)$, for $0 \leq i \leq 4$

$$G_{o_0}(\omega_u) = \frac{\theta_0}{1 - \theta_0}$$
$$G_{o_1}(\omega_u) = \frac{\theta_1}{(1 - \theta_0)^2}$$
$$G_{o_2}(\omega_u) = \frac{\theta_2}{(1 - \theta_0)^3} + \frac{20\theta_2}{(1 - \theta_0)^4}$$
$$G_{o_3}(\omega_u) = \frac{\theta_3}{(1 - \theta_0)^3} + \frac{60\theta_3}{(1 - \theta_0)^4} + \frac{60\theta_3}{(1 - \theta_0)^5}$$
$$G_{o_4}(\omega_u) = \frac{\theta_4}{(1 - \theta_0)^3} + \frac{60\theta_4}{(1 - \theta_0)^4} + \frac{360\theta_4}{(1 - \theta_0)^5} + \frac{240\theta_4}{(1 - \theta_0)^6}$$

Step 4: Calculate the variables $X_i$, for $0 \leq i \leq 4$

$$X_0 = \frac{G_{o_0}(\omega_u)}{\omega_u S_0}$$
$$X_1 = \frac{1}{\omega_u S_0} [G_{o_1}(\omega_u) - X_0 S_0 - \omega_u X_0 S_1]$$

Step 5: Calculate the variables $Z_i$, for $1 \leq i \leq 3$

$$Z_1 = X_1 + \omega_u X_2$$
$$Z_2 = X_1 + 3\omega_u X_2 + \omega_u^2 X_3$$
$$Z_3 = X_1 + 7\omega_u X_2 + 6\omega_u^2 X_3 + \omega_u^3 X_4$$

Step 6: Get a convenient numerical value of the parameter $\lambda$ by solving the following second order equation in the parameter $\lambda$

$$(Z_1^2 - X_1 Z_2)^2 + (Z_1 Z_2 - X_1 Z_3) \lambda + (Z_1^2 - X_1 Z_3) = 0$$

Step 7: Calculate the parameters $\mu, T_D, T_I$ et $K_e$

$$\mu = \frac{\lambda Z_1 + Z_2}{\lambda X_1 + Z_1}$$
$$T_D = \frac{(\lambda X_1 + Z_1) \omega_u^{1-\mu}}{[\mu(\mu + \mu)]}$$

$$T_I = -\frac{(\mu X_1 - Z_1) \omega_u^{1+\lambda}}{[\lambda(\lambda + \mu)]}$$

Two parameters $K$ and $T$ can be calculated directly from measured parameters $\beta, \nu$, and $\Omega$, as from section (II.B) we have:

$$K = -\frac{\delta}{J_D}$$

$$T = -\frac{1}{D}$$

These performance requirements of the projected feedback control system can be given in terms of the unity gain crossover frequency $\omega_u$ and the phase margin $\phi_m$ the two parameters $m$ and $\omega_u$ are obtained as:

- $\omega_u = \omega_q$ (the unity gain crossover frequency of the projected feedback control system)
- $m = 2[1 - (\phi_m/\pi)]$ ($\phi_m$ is the phase margin of the projected feedback control system)

Using the tuning algorithm of the fractional F'D' controller's five parameters given in [8] with some modification in the tuning procedure by using the plant parameter instead of its step response, we will get the following algorithm:

Remark: the extraction of all the formulas from (10) to (16) is presented in detail on Article [8].
online from equation (5) in function of three parameters \( \beta, \nu_m, \Omega_d \) which is measurable parameters, so we can represent two function as \( K = f(\beta, \nu_m, \Omega_d) \) and \( T = g(\beta, \nu_m, \Omega_d) \).

The global transfer function of plant includes actuator turbine blade adjustment is given by [15]:
\[
G_p(s) = \frac{K}{(1 + 0.25s)(1 + Ts)} = \frac{-2.5453}{(1 + 0.25s)(1 + 1.33s)}
\]

This transfer function represents the linearized model of wind turbine at specified point, the tow parameters K and T can calculate online such as: \( K = \frac{-\delta}{J_\Omega D} \) and \( T = \frac{-1}{D} \), so \( K = f(\beta, \nu_m, \Omega_d) \) and \( T = g(\beta, \nu_m, \Omega_d) \).

In this section we use proposed control strategy presented in figure (3) to control linear model of equation (20) with abrupt change in static gain and time constant. The dynamic performances requirements of the projected feedback control system are the unity gain crossover frequency are:
- \( \omega_c = 30 \) rad/sec.
- The phase margin \( \varphi_m = 70^\circ \)

The transfer function of the designed fractional PI\( \lambda \)D\( \mu \) controller through the reference model (8) \((m=1.22, \omega_c=30 \) rad/s) to satisfy the above requirements of the projected feedback control system is given by:
\[
C(s) = -14.7997 - \frac{8.0590}{s^{0.5781}} - 40.6346 s^{-0.5781}
\]

Figure (5) shows the bode diagram of open loop system. We can see the flatness of the phase around the crossover frequency \( \omega_c \), so we can say that we obtain a robust control.

In this controller design the uncertainties of the plant model and the high frequency noise were not taken into consideration. Yet, for practical problems, we need to use constraints on the sensitivity and complementary functions.

VI. SIMULATION RESULT

D. Control of linear system

From equation (7) we have the plant transfer function is given by:
\[
G(s) = \frac{K}{(1 + Ts)} = \frac{-2.5453}{(1 + 1.33s)}
\]
A. Control of Non linear model

Second step we use proposed control strategy of (figure 3) to control non linear system of equation (1) where wind turbine is coupled to Self-Excited Induction Generator (SEIG), the dynamic equation (3) is solved and $T_{SEIG}$ torque (which is considered as disturbance) is obtained from the model of the electric Generator which we have not presented in this paper for more detail see [15], so like it shown in figure (3) the parameters $K$ and $T$ has
been obtained online and the tuning algorithm evaluated as shown in figure 3. In order to show the closed loop behavior we work at variable speed operation so we suppose a change in wind speed between t=7s and t=12s (see figure 7.C) and we change set point in this range. Figure (7.C) show Turbine speed, at t=1s the change in time response illustrates the effect of the magnetization of electric machine; at t=3s a RL load is connected to the generator. The PI$\lambda$D$\mu$ parameters is shown by figure (7.E F) we can say that these parameters are located in a logical range. So the result is satisfactory and and the closed loop system behaves like the desired system although the plant’s parameters changing as shown by figure (7C). We note also that the change in the parameter $\lambda$ is less important compared to the change in $\mu$.

V. CONCLUSION

In this work we have proposed an extension of a new technique for tuning of PI$\lambda$D$\mu$ controller where the objective is to control a wind turbine speed; this plant has a complicated mathematical model and it needed to use a robust controller. The basic idea is to build a linear model with variable parameters in time, so the online implements of the tuning algorithm for the adaptive controller was proposed, this tuning procedure use the plant parameters that can be calculated online from measured parameters. The main advantage of this method is its simplicity to use where the controller parameters are calculates directly from the plant parameters and desired specification. Unfortunately, the user of fractional order operators has always a problem with implementation, in this work we use a recant variable order implementation which is very efficacy to use in adaptive control or system identification. The results obtained are very satisfactory and strategy of control can be used to control other types of dynamics system, also improvement can be done by changing the desired specification online.

REFERENCES


