Adaptive algorithms for target tracking

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Abstract—The quality of the tracking is greatly enhanced by a robust motion estimation. The objective is to develop a target tracking algorithm of a moving object, especially motion estimation of these. To realize the estimate, we chose stochastic filtering techniques. He concern the Kalman filter in the linear Gaussian, and sequential Monte Carlo methods in the nonlinear. Representation of state is permanently adapted according on current observations, to best represent the system dynamics. A comparison of the results given by the extended Kalman filter and particle filter is realized into simulation of the nonlinear systems target tracking.

Keywords—stochastic filtering, Kalman filter, Extended Kalman Filter, Monte Carlo method, Particle Filter, Target tracking.

I. Introduction

The target tracking is a discipline that covers a wide range of application, which extends robotic applications to military applications through civil aviation. Particular interest is worn for maneuvering target, seen the importance to track a target that tried to flee. The problem of target tracking is to estimate the kinematic parameters of a mobile (position, velocity, acceleration, ...) in sight of the measures noisy functions of these parameters (distance or angle) provided by a sensor (radar, sonar, optical sensor, ...). We are led to elaborate a filter that provides an estimate of the state at any time from harvested measures. This filter should be installed in a computer and the time of its execution must be reasonable. Recursive filters are then preferred. Indeed, for these filters, the estimated at the current time only depends on the current observation and the estimated of previous instant. The Kalman filter was developed for linear models Gaussian. In general, the problem of nonlinear filtering does not admit finite dimensional solution [1]. In most nonlinear case, we use versions of the Kalman filter said extended (EKF) that makes the linear dynamic model around an approximate solution. When the system is highly nonlinear and / or large, the extended Kalman filter can diverge. Monte Carlo methods are proposed to solve the nonlinear filtering problem, they are based on the law of large numbers and have performances few sensitive to the size of the state space. These methods can estimate the conditional density in real time even in large dimension.

Particle methods are sequential version of the Monte Carlo methods to solve the filtering. However, the particle filter described has a major defect, the weight of the particles tend to degenerate. So the particle system can not correctly represent the conditional density and the filter diverges. To avoid this phenomena, a new step called resampling was introduced. A comparison between the extended Kalman filter and the particle filter in the nonlinear systems in target tracking.

II. Description of filtering problem

The filtering problem is to determine the estimators of the variables of a dynamic system (state variables) subject to disturbances and observed partially. The modeling of the evolution over time of the states \( \{X_k\}_{k \geq 0} \) of the system considered is then written in the form of a deterministic evolution part and a stochastic part,

\[
X_t = X_0 + \int_0^t f(s, X_s, w_s)ds
\]  

(1)

where \( X_0 \) is a given law, \( f \) is a deterministic function characterizing the dynamics and \( \{w_t\}_{t \geq 0} \) is a standard Wiener process modeling the random perturbations of the dynamics and our ignorance of the model. The process \( X, \) \( (1) \) is a Markov process, we denoted by \( Q_k \) its transition nucleus. In order to determine the state of the system, it is necessary to construct an observation equation that links at instants \( k \) observations \( Y_k \) to the current state \( X_k \). These measures are flawed due to the imperfection of the sensor. \( \{Y_k\}_{k \geq 0} \) observations is modeled by the following equation,

\[
Y_k = h_k(X_k) + v_k
\]  

(2)

Where \( h_k \) is the known function of observing and \( \{v_k\}_{k \geq 0} \) a sequence of random variable (r.v) with statistical known modeling the imperfect observations. We seek to determine the system state \( X_k \) from noisy observations \( \{Y_1, ..., Y_k\} \). The choice of criteria describing the precision of the estimated is not unique. In all cases, the richest information available given the observations \( \{Y_1, ..., Y_k\} \) is contained in the conditional distribution of \( X_k \) knowing the past observations. With the data of this law, we are free in the choice of the estimator. The filtering problem will be to calculate (exact or approximate way) this conditional probability.
The study is to be interested in target tracking application of nonlinear filtering. In tracking, the system is a mobile (e.g. a ship or aircraft) which is to be estimated at each instant \( k \), the state \( X_k \), constituted by its position and speed. If the target is not in phase for maneuver, it is reasonable to choose a dynamic model rectilinear uniform a priori. A radar provides noisy measurements \((Y_k)_{k \geq 1}\) of the distance to the plane and the angle formed by a reference axis and the sighting axis. The particular difficulty of this application is the real time constraint, that is to say, the computation time of the estimate provided by the filter should be of the order of the measurement period \((k + 1)T - kT\).

A. The optimal filter

Optimal filtering is to calculate the conditional density \( p_k \) of the state \( X_k \), knowing the observations \( Y_{\leq k} = (Y_1, ..., Y_k) \) up to the current time \( k \).

\[
p_k(dx) = P[X_k \in dx | Y_{\leq k}]
\]

Following the conditional probability distributions \((p_k)_k \geq 0\), is called optimal filter. In practice, it is desired to obtain a recursive algorithm \( P_k \). The calculation of \( P_k \) should be a function only of the last observation \( Y_k \) and the previous conditional distribution \( p_{k-1} \). The effective calculation of the optimal filter is a very delicate problem, by against his mathematical formalism is easy.

The passage from \( p_{k-1} \) to \( P_k \) involves the conditional law of \( P_{k|k-1} \) of the state \( X_k \) knowing the observations \( X_{\leq k-1} = (y_1, ..., y_{k-1}) \) until the previous time \( k-1 \).

\[
p_{k-1}(dx) = P[X_k \in dx | Y_{1:k-1}]
\]

Following the laws of probability \((p_k|k-1)_{k \geq 0}\), is called prediction filter. The evolution of the optimal filter comprises two essential steps.

**Prediction:** The optimal filter predicting step uses a priori knowledge of the system, through the transition nucleus \( Q_k \), in realizing the transition from \( p_{k-1} \) to \( P_{k|k-1} \) as follows,

\[
p_{k-1} \equiv \int R dx' Q_k(dx',dx) = (Q_k p_{k-1})(dx)
\]

Where \( R^d \) denotes the state space. The prediction step is linear.

**Correction:** The second step uses the observation \( Y_k \), through the conditional density of \( Y_k \) knowing \( X_k \). This density denoted \( g_k(X) = p(Y_k/X_k) \) is called likelihood, it corrects the predicted density \( p_{k|k-1} \), by applying the formula of Bayes:

\[
p_k(dx) = \frac{g_k(x) p_{k|k-1}(dx)}{\int_{\mathbb{R}^T} g_k(x, x') p_{k|k-1}(dx')} \tag{4}
\]

The correction step is not linear.

\[
p(X_{k-1}|Y_{\leq k-1}) \xrightarrow{\text{prediction}} p(X_k|Y_{\leq k-1}) \xrightarrow{\text{correction}} p(X_k|Y_k)
\]

Fig. 1 gives the classic scheme of the decomposition of an iteration of a nonlinear optimal filter in a prediction step, involving the Transition law, and a correction step involving the new observation and local likelihood function associated.

The main classical methods proposed to solve the filtering problem are grouped into two classes: analytical methods, namely the Kalman filter and its derivatives, and digital methods.

III. Analytical Methods

It is particularly interested in Kalman-Bucy filters and extended Kalman. These are mostly used to solve linear and nonlinear filtering problems [11].

A. Kalman-Bucy filter (KF)

The Kalman filter is an essential tool for solving filtering problems, it mainly concerns linear systems [18]. The filter was developed by Kalman [2] in 1960 for the discrete case and resumed in 1961 by Kalman-Bucy [3] to the continuous case.

B. Extended Kalman Filter (EKF)

This time, the dynamic system is nonlinear with white Gaussian additive noise, independent of each other and independent of the original law [12].

\[
\begin{align*}
X_k &= f_k(X_{k-1}) + W_k \\
Y_k &= h_k(X_k) + V_k
\end{align*}
\]

Where \( f_k \) and \( h_k \) functions are nonlinear. The optimal filter associated with this system is no longer Gaussian and it can not solve explicitly as in the linear / Gaussian case. A natural idea is to linearize the system dynamics around the predicted state \( X_{k|k-1} \), and around the current state \( X_k \), then apply the Kalman filter technique.
For the prediction phase, state equation is linearized around \( \hat{x}_{k-1} \):

\[
X_k \equiv F_k X_{k-1} + F_{k-1} \hat{X}_{k-1} + W_k \quad \text{avec} \quad F_k = \nabla f_k (\hat{x}_{k-1})
\]

\[
\hat{x}_{k|k-1} = f_k (\hat{x}_{k-1})
\]

\[
P_{k|k-1} \equiv F_k^T P_{k-1} F_k + Q_k
\]

For correction phase, observation equation is linearized around the estimated current \( x_{4k-1} \), obtained:

\[
Y_k \equiv H_k X_k + h_k (\hat{x}_{4k-1}) - H_k \hat{x}_{4k-1} + V_k \quad \text{avec} \quad H_k = \nabla h_k (\hat{x}_{4k-1})
\]

\[
K_k = P_{k|k-1} H_k^T \left( H_k P_{k|k-1} H_k^T + R_k \right)^{-1}
\]

\[
\hat{x}_k = \hat{x}_{4k-1} + K_k \left[ Y_k - h_k (\hat{x}_{4k-1}) \right]
\]

\[
P_k = \left( I - K_k H_k \right) P_{k|k-1}
\]

In practice, when the linear system is a bad approximation due to non linearity too large, the extended Kalman filter tends to diverge, giving too much trust in the linear system. To remedy this, it was proposed to increase artificially, either the covariance of the state noise in order to compensate for the non linearity too large, the extended Kalman filter tends to diverge, giving too much trust in the linear system. To remedy this, it was proposed to increase artificially, either the covariance of the state noise in order to compensate for the fact that the prediction model is not suitable, either to increase the covariance \( P_{4k-1} \), thereby giving less importance to the past. Although it is used for about thirty years, theoretical studies to justify were developed only very recently, essentially through the work of Picard [4]. This work realized for the continuous-time state model / continuous-time observation model justify the use of EKF algorithm when the observation is very good and when the observation noise, the initial error are low. These results were extended by Getout-Petit [5] in some cases where the observation is partially very good, typically when a coordinate is completely observed. However, EKF can not apply when the conditional density is multimodal.

IV. Numerical Methods

In the context beyond the three-dimension, the Monte Carlo methods seem an interesting alternative to treat the nonlinear filtering problem.

V. Sequential Monte Carlo Methods : particle filters

Particle methods are based on the principle of Monte Carlo to estimate and predict online dynamic systems. They enable to approach the solution of a nonlinear filtering problem in discrete time. The method consists of an importance sequential sampling with resampling [13].

A. Sampling importance sequential Bayesian (S.I.S) :

The Monte Carlo approach to nonlinear filtering problem in discrete time is to approach with samples the filtering law \( p(X_i | Y_{i-1}) \).

The problem is to approach the distribution of trajectories \( p(X_{0:4}|Y_{1:4}) \). The sequential resolution of this problem is to solve the filtering problem by considering only the extremities of the trajectories [14].

The importance sampling method [6] can be applied, using an importance function \( q(x_{0:4}|Y_{1:4}) \). To allow sequential importance sampling progressively the arrival of data, the importance function may be chosen so that

\[
q(x_{0:4}|Y_{1:4}) = \frac{1}{\prod_{i=1}^{T} q(x_i|x_{0:i-1},y_i)}
\]

Using Bayes' theorem and independence property [7] [16], the law \( p(x_{0:4}|Y_{1:4}) \) is expressed in functions of \( p(y_i|x_i) \), \( p(x_i|x_{i-1}) \) and \( p(x_{0:4}|y_{1:4}) \).

\[
p(x_{0:4}|Y_{1:4}) \propto \int p(y_i|x_i) p(x_i|x_{i-1}) p(x_{0:4}|Y_{1:4})
\]

It is then possible to construct a recursive formula for calculation importance weight:

\[
w_i^k = \frac{p(x_{0:4}|Y_{1:4})}{q(x_{0:4}|Y_{1:4})}
\]

\[
w_i^k \propto w_{i-1}^k \frac{p(y_i|x_i) p(x_i|x_{i-1})}{q(x_i|x_{0:i-1},y_i)}
\]

The resolution of the filtering problem can finally be done recursively, with the following two steps of prediction and correction :

- Prediction Step

\[
X_i \sim q(x_i|x_{0:i-1},Y_{1:4}), \quad i = 1 : N
\]

- Correction Step

\[
w_i^k \propto w_{i-1}^k \frac{p(y_i|x_i) p(x_i|x_{i-1})}{q(x_i|x_{0:i-1},Y_{1:4})}, \quad i = 1 : N \quad \text{et} \quad \sum_{i=1}^{N} w_i^k = 1
\]

The estimate of the law \( p(x_{0:4}|Y_{1:4}) \) The estimate of the law \( X_{i} \) is expressed sequentially. Each trajectory \( X_{0:i-1} \) is prolonged by a new state \( X_i \) drawn according to importance
filters particulate
the basis of the sequential Monte Carlo methods also called
particles due to the inevitable increase in variance
particular a problem of degeneration of the weight of the
suffers of the limitations importance sampling and in
algorithm is commonly known by the acronym
rapidly approach zero
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in its raw version soon led to a bad estimate of the filtering

It is known that the use of the sequential importance sampling
such as the effective number of particles :

The selection will be to eliminate the most weakly weighted
importances. The degeneration of the weight is detected when
If this number is near to N the particles are of equal
proportionnally to weight

By defining a generic structure based on the S.L.S algorithm, in
which a step of resampling is added, the resulting
algorithm is commonly known by the acronym S.I.R, and is
the basis of the sequential Monte Carlo methods also called
filters particulate [10].

This simple estimation algorithm of the filtering Law
p(X_{i|k}|Y_{1:k}) has the advantage of being parallelizable [8].
However, its recursive nature raises some problems. He
suffers of the limitations importance sampling and in
particular a problem of degeneration of the weight of the
particles due to the inevitable increase in variance [9].

B. Degeneration and resampling

It is known that the use of the sequential importance sampling
in its raw version soon led to a bad estimate of the filtering
distribution. In practice, we observe an increase in the
variance of weight in time and all weights except one will
rapidly approach zero [9]. Ideally, all trajectories should have
equal importance in the approximation.
The use of resampling methods avoids this problem. The idea
is to change the current set of samples using a selection step.
The selection will be to eliminate the most weakly weighted
samples and duplicate the samples of strong weight. You can
decide if resampling is necessary or not relying on a criterion
such as the effective number of particles :

If this number is near to N the particles are of equal
importances. The degeneration of the weight is detected when
N_{eff} become weak.
The technique of classical resampling consists in a draw with
replacement among \(X_{i|k}^{'}\) proportional to weight
\(w_{i}^{'}\). This approach, known as multinomial resampling,
ensures that the number of descendant \(N_{i}^{'}\) of each particle is
proportional to its weight (i.e. \(E(N_{i}^{'}) = N w_{i}^{'}\)).

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\[ p(x_{i|k}^{'}|y_{1:k}) = \sum_{i=1}^{N} w_{i}^{'} \delta(x_{0:k}^{'} - x_{i|k}^{'} ) \quad (10) \]

Or filtering distribution \( p(x_{i|k}^{'}|y_{1:k}) : \)

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filters particulate [10].

A displacement is a spatial transformation that results from a
rotation and a translation. Any movement may be decomposed
into two elementary transformations. The analytical formalism
of transformation brings extensive knowledge of the nature of
the complexity of tracking problems.
We can easily guess that follow a trajectory to its terminal
member presupposes the mastery of the coordination of all
degrees of freedom, in position, velocity and acceleration [15]
[17].

VII. Application to the tracking problem

The problem of target tracking is to estimate the kinematic
parameters of a mobile (position, velocity, acceleration, ...)
from noisy measurements based on these parameters provided
by a sensor (radar, sonar, optical sensor, ...). The dynamic
model of the target is often linear :

\[ X_{k} = F_{k} X_{k-1} + W_{k} \quad (13) \]

In return, the observation equation tying the measure to the
state is often non-linear and depends on the sensor used
(measurement of distance, angle or Doppler).

\[ Y_{k} = h_{k}(X_{k}) + V_{k} \quad (14) \]

\(W_{k}\) and \(V_{k}\) are Gaussian white noise. We find exactly the
terms of a classical filtering problem.

We interested on the simulated behavior of the particle
methods on concrete problems tracking, showing the
contribution of particle methods face a classical methods such
as EKF, which are sometimes setting in default. We compare
the performance of EKF and PF on the tracking problem by
measure distance and angle.

\[
\begin{bmatrix}
    X(t_{1,i}) \\
    X(t_{2,i}) \\
    Y(t_{1,i}) \\
    Y(t_{2,i})
\end{bmatrix} =
\begin{bmatrix}
    1 & T & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & T \\
    0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    X(t_{1,i}) \\
    X(t_{2,i}) \\
    Y(t_{1,i}) \\
    Y(t_{2,i})
\end{bmatrix} +
\begin{bmatrix}
    T/2 & 0 \\
    T/2 & 0 \\
    0 & T/2 \\
    0 & T/2
\end{bmatrix}
\begin{bmatrix}
    w_{1}^{2} \\
    w_{2}^{2}
\end{bmatrix}
\]

V. Trajectory modeling

fig.2. Evolution of the conditional density
We consider the tracking problem of a mobile moving in a plane supplied with a Cartesian coordinate reference. The evolution of the mobile is well described by the evolution of its coordinates \((X(t), Y(t))\). The discrete state equation \(X_k\) is given as follows.

Where \(T = t_{k+1} - t_k\) is the discretization period of the model. For whole number \(k \geq 0\), there is

\[
X_k = \begin{bmatrix} X_{1k} \\ X_{2k} \\ X_{3k} \\ X_{4k} \end{bmatrix} = \begin{bmatrix} X(t_k) \\ \dot{X}(t_k) \\ Y(t_k) \\ \dot{Y}(t_k) \end{bmatrix}
\]

\(X_k\) form the position and velocity following each of the coordinates. It is a problem of four dimension.

### A. Simulations

The radar provides noisy measurements of the distance between the observer and the mobile, as well as the angle formed by the vertical axis and the sight line, i.e.,

\[
h(X_k) = \begin{bmatrix} \sqrt{X_{1k}^2 + X_{3k}^2} \\ \arctan(X_{3k}/X_{1k}) \end{bmatrix}
\]

We consider the following problem of tracking by measuring distance and angle.

- The initial state is \(X_0 = (5km, -20m/s, 5km, 20m/s)\) with as covariance matrix \(P_0 = \text{diag}[2km^2, 0.2, 2km^2, 2km^2]\). The dynamics of the target is noisy according to equation (15) with a noise on the acceleration \(w_1 = w_2 = c\) fluctuating.
- The time of tracking is \(T = 200s\) with a measurement period equal to one.
- The standard deviation of the measurement noises on the distance and angle are respectively 50m et 1 degree.

The number of particles used by the PF varies between \(N = 100\) and \(N = 1500\) particles.

We underline that the conditions guaranteeing good behavior of EKF are not verified even if the measures are very precise.

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fig.3. La trajectoire vraie, estimée de la cible par l'EKF

fig.4. La position vraie, estimée suivant l'axe X et l'axe Y par l'EKF

fig.5. L'erreur entre X et Xest, Y et Yest.

fig.6. L'angle & distance vraies, estimées par SIR adaptatif avec \(N = 500\) et \(N_{thresh} = 0.5\ N\)

fig.7. L'angle & distance vraie, estimées par SIR systématique avec \(N = 500\)
The extended Kalman filter set some time to follow the trajectory, because the starting conditions are badly initialized. From $t = 400$ s the filter begins to diverge because of the strong nonlinearity. The dynamic noise plays a role in the filter divergence.

We note that the SIR filter estimates better the trajectory of the target compared to that of the EKF.

We constate on the fig. 7 of systematic SIR filter, that the particle cloud is impoverished compared to fig. 6 of SIR adaptive filter. So the estimated given by the systematic SIR is less reliable. The computation time for the systematic SIR is greater than adaptive SIR.

The particle filter with adaptive resampling gives good result compared to the particle filter with systematic resampling.

Also, we get good results, increasing the number of samples. Similarly, we remark the influence of the threshold value on the result of SIR adaptive filters. more the threshold is small, more the results are better. The number of samples plays a primordial role in the estimation. The EKF rarely that it gives the result of SIR adaptive filters. more the threshold is small, the results are better. The number of samples plays a primordial role in the estimation. The EKF rarely that it gives the result of SIR adaptive filters.

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VIII. Conclusion

The nonlinear filtering research is strongly motivated by many applications. In this research we are interested at the Bayesian estimation of the Markov state process $(X_k)_{k=0,N}$ observed through the process of measures $(Y_k)_{k=1,N}$. Particularly, the objective was to provide an estimate of the conditional distribution $p(X_k | Y_{1:k})$. If the optimal filter provides a recursive expression of this law, it is computable in practice only in very limited circumstances, such as the linear case with Gaussian additive noise.

Monte Carlo methods were permit to develop notably in the context of filtering a class of very effective methods, denominated sequential Monte Carlo methods. At every moment, the law $p(X_k | Y_{1:k})$ is approximated by a weighted (importance) sum of Dirac laws centered in the particles. These particles and their weights evolve over time according to a sequential importance sampling, and are resampled to keep only the best in the sense of likelihood of observations. These methods are theoretically validated by the existence of probabilistic theorems ensuring notably uniform convergence to time of the particulate solution under mixing conditions.

Finally, we have presented an application of the particle filter that we have supplied for tracking a single target observed through measurements of distance and angle. We have shown the practical advantage of adaptive resampling from retarding degeneration of weight relative to the systematic resampling. We have also proved his superiority in front of an extended Kalman filter.

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