Study of the Effet of the Geometry Shape of defect on the Reliability of NDT Device

Z. Oudni
Numerical Modeling of Electromagnetic Phenomena and Components Laboratory
Mouloud Mammeri University,Tizi Ouzou, Algeria
z_mohellebi@yahoo.fr

H. Mohellebi,
Numerical Modeling of Electromagnetic Phenomena and Components Laboratory
Mouloud Mammeri University,Tizi Ouzou, Algeria
mohellebi@yahoo.fr

Abstract— This work concerns the study the effect of random properties and defect geometry on the reliability analysis of the non-destructive testing inspection device. Two kinds of defects geometries are considered as rectangle and triangle shapes respectively. A stochastic finite element method (SFEM) was used to solve the 2D electromagnetic equation in cylindrical structure. The results obtained for both rectangle and triangle random shapes are given.

Keywords— ***random geometry, random propertie, nondestructive testing; reiability analysis, stochastic finite elements.***

I. INTRODUCTION

The study of practical systems as electromagnetic usually needs to know the data input to obtain the output information. In many case physical properties such as electrical conductivity, magnetic permeability and electrical permittivity must be known at the beginning of problem treatment [1]-[2]-[3]-[4]. The defects shapes are also needed in the inspection process. In this situation input data are often used with some uncertainty[5].
The stochastic finite element method used has some advantages especially in the study and analysis of sensitivity of the system in a single step [5]-[6]. The present work concerns the study of the reliability of non destructive testing device when the defect concerns the uncertainty on the electrical conductivity [7]-[8]-[9] and the defect shape (rectangle and triangle) which are considered as random variable and are developed in series of Hermite polynomials. A finite element code is developed under Matlab package considering a nondestructive system.

II. RANDOM GEOMETRY DEFINITION

In the case of random geometry the two random variables G_h and G_w of chaos polynomial of order 2 is expressed as following:

$$\begin{bmatrix} G_h \\ G_w \end{bmatrix} = \begin{bmatrix} \sim_{01} & \sim_{11} & 0 & \sim_{12} & 0 & 0 \\ \sim_{02} & 0 & \sim_{12} & 0 & 0 & \sim_{22} \end{bmatrix} \begin{bmatrix} 1 \\ \P_1 \\ \P_2 \\ \P_1^2 \\ \P_1\P_2 \\ \P_2^2 \end{bmatrix} \qquad (1)$$

$\sim_{01}, \sim_{11}, \sim_{12}$ and $\sim_{02}, \sim_{12}, \sim_{22}$ coefficients are obtained from (8), (9) and (10) for each random variable G_h and G_w. In our development the random variables G_h and G_w are the height and the width of the defect respectively.
The projection of the defects dimensions, the height and the width, G_h and G_w respectively in the basis of Hermite polynomials leads to [1]-[3]-[6]:

$$G_h = \sum_{i=1}^{n_A} h_i H_i(\P_1, \ldots, \P_M) \qquad (2)$$

$$G_w = \sum_{i=1}^{n_A} w_i H_i(\P_1, \ldots, \P_2) \qquad (3)$$

h_i and w_i are the random coefficients of defect geometry to be searched.

II.1. HERMITE POLYNOMIALS

Let consider the random variable *Y*, having as input parameters the vector ξ of *M* independent stochastic variables It can be shown that if *Y* has a finite variance then *Y* can be written as a linear combination of multivariate polynomials on the basis of the Hermite polynomials [1]-[3].

$$Y = \sum_{i=1}^{n_A} \sim_i H_i(\P_1, \ldots, \P_M) \qquad (4)$$

$\{H_i\ (Y),\ i=0,\ldots,\ \infty\}$ are Hermite polynomials, $\{\sim_i,\ i = 0,\ldots,\ \infty\}$ are coefficients of Hermite polynomials where ξ is a reduced centered Gaussian random variable (r.c.g.r.v) with the following density of probability [1]-[3]-[6]:

$$¤(G) = \frac{1}{\sqrt{2®}} e^{\frac{-G^2}{2}} \qquad (5)$$

In (4) $\sim_i$ are unknown coefficients to be determined. We notice that all Hermite polynomials are orthogonal with regard to the Gaussian measure according to the orthogonally property:

$$E\left[H_n(¶(^-))H_m(¶(^-))\right] = 0 \qquad if \quad n \neq m \qquad (6)$$

E [.] denotes the mathematical expectation.

For the computation of the coefficients of Hermite polynomials, the work develops the method of projection. Thus, the coefficients of Hermite polynomials are given by the following expression [1]-[3]-[4]-[5]:

$$\sim_i = \frac{E\left[GH_i(¶)\right]}{i!} \qquad (7)$$

The identification of the Hermite polynomial coefficients in the case of iso-probabilistic transformation is performed using the next formulae:

$$\sim_i = \frac{1}{i!}\int_R F_G^{-1}(\Phi(t))H_i(t)¤(t)dt \qquad (8)$$

When G follows a Gaussian law of average $«_t$ and of standard deviation $±_t$, the calculation of the coefficients of the development (7) is as follows:

$$\sim_0 = «_t \qquad (9)$$

$$\sim_1 = ±_t \qquad (10)$$

$$\sim_i = 0 \qquad \text{if} \quad i \phi 1 \qquad (11)$$

The projection of the unknown A in the basis of Hermite polynomials leads to [1]-[3]-[6]:

$$A = \sum_{i=1}^{n_A} A_j H_j(¶_1,...,¶_M) \qquad (12)$$

$$± = \sum_{i=1}^{n_A} ±_i H_i(¶_1,...,¶_M) \qquad (13)$$

II.3. STOCHASTIC FINITE ELEMENTS FORMULATION

II.3.1. STOCHASTIC LINEAR SYSTEM CONSTRUCTION

The stochastic finite element linear system is constructed after the projection in the Hermite polynomials basis of the random variables with the combination of equations (12), (13).

The non-destructive testing of interest problem is built with the consideration that the source current density and the electrical conductivity of the medium are given.

The minimization when using Galerkin method means requiring the orthogonality of the residue with the base of projection functions {Ψ_j, j=0.., p}: By taking as test function $\sim_i$ the chaos polynomial Ψ_j. This leads to[1]-[3]:

$$\sum_{j=0}^{p-1} M_0 A_j E\left[\cdot_j \cdot_k\right] + j\mu \sum_{i=0}^{p-1}\sum_{j=0}^{p-1} N_i A_j E\left[\cdot_i \cdot_j \cdot_k\right] = F_k \qquad (14)$$

$k = 0,..., p-1$ and $n_A = p-1$:

The final form of the stochastic finite element and harmonic linear system is:

$$\sum_{j=0}^{p-1} (M^s{}'_{jk} + j\mu N^s{}_{jk})A_j = F^s{}_k \qquad (15)$$

$$M^s{}'_{jk} = d_{0jk} M_0 \qquad (16)$$

$$N^s{}_{jk} = \sum_{i=0}^{p-1} d_{ijk} N^s{}_i \qquad (17)$$

$$d_{ijk} = \begin{cases} \dfrac{i!j!k!}{\left(\frac{i+j-k}{2}\right)!\left(\frac{j+k-i}{2}\right)!\left(\frac{k+i-j}{2}\right)!} & if \begin{cases} (i+j+k) \ even \\ k \in [|i-j|, i+j] \end{cases} \\ 0 & otherwise \end{cases} \qquad (18)$$

M^s, N^s, F^s are the random linear matrixes and source vector respectively related to solving problem.

The system obtained from (18) is:

$$\begin{bmatrix} P^s{}_{00} & P^s{}_{01} & P^s{}_{12} \\ P^s{}_{01} & P^s{}_{11} & P^s{}_{21} \\ P^s{}_{02} & P^s{}_{12} & P^s{}_{22} \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ F_1 \\ F_2 \end{bmatrix} \qquad (19)$$

The matrix entries on the right hand side of (19) are obtained using the relations (16), (17) and the expression (20):

$$P^s = M^s{}'_{jk} + j\mu N^s{}_{jk} \qquad (20)$$

Where A_0, A_1, A_2 are the solutions of the stochastic complex algebraic system and F_0, F_1, F_2 are the source vector components.

III. Application and Results

The study was conducted with the assumption of 2D axisymmetric problem geometry as shown in Figure 1 [11]. The defect realised on the conducting plate of 1 M S/m is characterised by 10 mm length, 0.22 mm width and 0.75 mm depth. The inductor representing the sensor has 140 coil turns and supplied by alternative current of amplitude 0.008 A with a frequency of 150 kHz. The results obtained are shown by Fig.3, Fig.4 and Fig. 5 for magnetic vector potential distribution A_0, A_1 , A_2. In Fig.6 for impedance variation in case of rectangle defect shape, and in Fig.7 the reliability index [10] for both rectangle and triangle shapes.

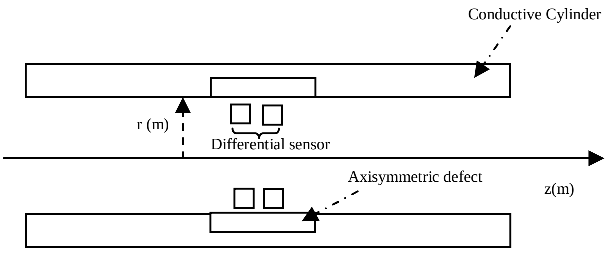

Fig. 1. 2D axisymmetric NDT device

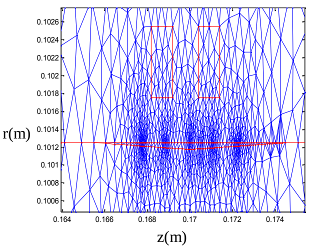

Fig. 2. Mesh of solving domain

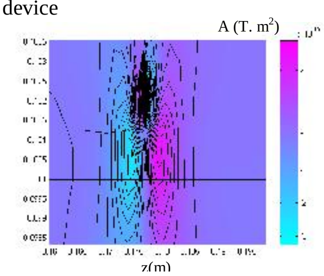

Fig.3. Magnetic vector potential distribution A_0

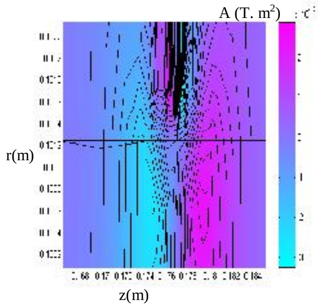

Fig.4. Magnetic vector potential distribution A_1

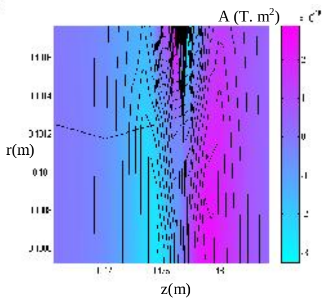

Fig.5. Magnetic vector potential distribution A_2

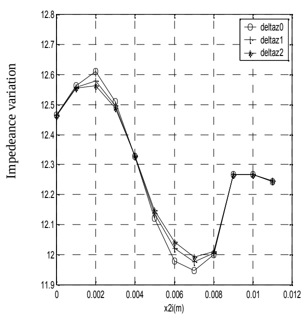

Fig.6. Impedance variation for triangle defects

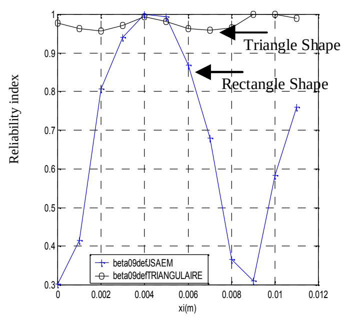

Fig.7. Reliability index (b) for triangle and rectangle defect shape

The results obtained for the reliability index in case of triangular defect are between 3.35 and 3.50 (figure7) and then the probability of failure is then very small. But when the defect has a rectangular form, the reliability index is very important and then yields to an important value of the probability of failure.

IV. Conclusion

The present work presents the development of the intrusive spectral finite element method in case of random geometry. The application considered is a non destructive testing device with a defect for the reliability analysis. Two kind's geometries shapes of defects are considered: a triangle and a rectangle. For Booth cases, the reliability index was computed using the developed intrusive spectral stochastic finite element method. The results obtained show that the triangle shape defect induces a very small probability of failure value comparatively to the one of rectangle shape form

References

[1] M Bervellier, ''Eléments finis stochastiques: Approche intrusive et non intrusive pour des analyses de fiabilité'', Thèse de doctorat, Université Blaise Pascal-Clermont II, 2005.

[2] Z. Oudni, M. Féliachi, H. Mohellebi,'' Stochastic Finite Elements Method Applied to Magnetic Levitation Problem with Random Magnetic Permeability,'' ISEF'2011, Madeira Island, Portugal, 1-3 September, 2011.

[3] R. G. Ghanem, P. D. Spanos, *Stochastic finite elements- A spectral approach*, (Springer, Berlin, 1991).

[4] M. Lemaire**,** in Proceedings of the 15th European Conference on Numerical Methods in Electromagnetism, Lille, 2006, p. 3

[5] H. Mac, S. Clénet, J. C. Mipo, O. Moreau, IEEE Trans. Magn. **46**, 3385(2010).

[6] R. Gaignaire S. Clenet, B. Sudret , O. Moreau, IEEE Trans. Magn. **43**, (1209) 2007.

[7] K. Beddek, S. Clenet, O. Moreau, V. Costan, Y. Le Menach, A. Benabou, IEEE Trans. Magn. **48**, 759 (2012)

[8] Y. Le Bihan, J. Pavo, C. Marchand, Eur. Phys. J. Appl. Phys, **43**, 213 (2008)

[9] C. R. A. Da Silva JR, A. T. Beck, E. Da Rosa, Lat. Amer. J. Soli. Stru., **6**, 51 (2009)

[10] Thomas, J. L., *Report*, ESA IGELEC, Nantes University, France, 1998