Non intrusive stochastic finite elements method applied for non destructives NDT problems

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Abstract—The current work concerns the study of harmful defect detection for the inspected structure by eddy current non destructive testing. So a non intrusive stochastic finite element method was exploited in the case of 2D magnetodynamic equation. The electrical conductivity is considered as random variables and generated using Monte Carlo method.

Keywords— Stochastic finite element method; Eddy current ;Conducting plate; Random variable

I. INTRODUCTION

Since the 1950s has been an increase in the use of Non-destructive testing (NDT) techniques which rely on the application of physical principle [6]. Non-destructive testing plays an important role in the quality control not only of the finished products, but also of half finished products as well as the initial raw materials. It can be used at all stages of the production process during the process of establishing a new technology by product quality or when developing a new product. Outside the manufacturing field, NDT is also widely used for routine or periodic control of various items during operation to ascertain that their quality has not deteriorated with use [9][5].

Various methods are used, such as ultrasonic, infrared, radiographic, optical and Eddy current [5][7].

Eddy current testing has developed increasingly in the testing of materials, especially in the aircraft and nuclear industries [1][2]. The extensive research and development in highly sensitive eddy current sensors and instruments indicates that eddy current testing is currently a widely used inspection technique. In other meaning eddy current testing is one of the most extensively used non-destructive techniques for inspecting electrically conductive materials at very high speeds that does not require any contact between the test piece and the sensor[5] [13].

The finite element method is currently used to examine electromagnetic devices in the nonintrusive method two approaches are classically distinguished: The projection method and the regression method [12]

II. DESCRIPTION OF THE PROBLEM

In this work we propose to study a conductive plate having a crack along the x coordinate by eddy current non-destructive testing s when the defect is represented as a change of physical property we precise that the electrical conductivity is considered as a random variable which is generated using Monte Carlo simulation.

The relevant configuration is shown schematically in figure 1.

![Geometry of the studied problem.](image)

We consider three cases of conductivity:

- Increasing conductivity
- Decreasing conductivity
- Any conductivity

We also calculate the impedance and the resistance to make a comparison and the results obtained are given in Section 4. The resolution of the problem is made using computer modeling.
III. FINITE ELEMENT FORMULATION

The study of Eddy current problem allows considering the 2D electromagnetic equation given in the (x,y) plan.[11]:

\[-\frac{\partial}{\partial x}\left(\mu \frac{\partial A_x}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial A_y}{\partial y}\right) + j\sigma \omega A_z = j z\ (1)\]

With \( A_x = A_2 \vec{k} \) and \( J = J_{zz} \vec{k} \)

\( \sigma \) is the electrical conductivity \((\Omega.m)^{-1}\) depending on material area. It is considered as random variable in the defect zone and noted \( \sigma_d \). \( \mu \) is the magnetic permeability \((H.m)^{-1}\). \( \omega = 2\pi f \), where \( f \) represents the frequency of feeding (Hz), \( J_{zz} \) is the source current density component in z direction.

Using the Green theorem and imposing boundary conditions of Dirichlet type on the boundary of the studied domain we obtain the following equation:

\[ \int_{\Omega} \nabla \vec{A}_j \cdot \nabla \vec{A}_j d\Omega + j\sigma \int_{\Omega} \vec{A}_j A_z d\Omega d\xi = \int_{\Omega} J_{zz} \vec{A}_j d\Omega d\xi \ (3)\]

\( A_x = \sum_{i=1}^{n} \alpha_i A_i \)

\( \alpha_i \) is the projection function, \( \alpha_i \) is the shape function associated with the node j. After discretization and assembly we obtain the following system:

\[ [[M] + j\sigma \omega [N]] [\bar{A}] = [\bar{F}] \ (4)\]

\( [M] \) is the stiffness matrix, \( [N] \) is the dynamic matrix, \( [F] \) is the source vector and \( [A] \) is the unknowns vector.

IV. GLOBAL PARAMETERS

A. The impedance\((Z)\)

The impedance computation is derived using a general method based on the evaluation of the magnetic flux as indicated in the equations below [10]:

\[ Re(\Omega) = -\frac{N^2}{J^2} \omega \int_{s} 2\pi r \text{Im}(A) \ ds \]

\[ Im(\Omega) = -\frac{N^2}{J^2} \omega \int_{s} 2\pi r \text{Re}(A) \ ds \]

Re\((A)\) and Im\((A)\) are the real and imaginary part of magnetic vector potential respectively, \( N \): coil numbers and \( S \): surface coil, \( r \): inductor radius.

B. The power density

The power density \( P \) is computed in the defect zone and given by:

\[ P = \int_{\Omega} \frac{1}{2} \sigma \omega^2 A^2 d\Omega d\xi \]

C. The current

The total current \( I \) in the load piece in obtained using the expression below:

\[ I = \int_{\Omega} -j \sigma \omega A d\Omega d\xi \]

V. APPLICATION AND RESULTS

The 2D geometry of the problem is shown in figure 1 with the associated dimensions [11]. The physical parameters are: the sensor is supplied by alternative current of amplitude 0.008 A and a frequency about of 100 kHz, a number of turns is 250.

The results obtained, using non intrusive stochastic finite elements method [3], are given in figures 2-7.

A. For increasing conductivity

Fig 2. The electrical conductivity variation along surface defect for increasing random distribution

Fig 3. The power density variation along surface defect for increasing random distribution of electrical conductivity
Fig 4. The resistance variation along surface defect for increasing random distribution of electrical conductivity

B. For decreasing conductivity

Fig 5. The electrical conductivity variation along surface defect for decreasing random distribution

Fig 6. The resistance variation along surface defect for decreasing random distribution of electrical conductivity

Fig 7. The impedance variation according to sensor position.

This result indicates the variation of the impedance for the three supposed kind of electrical conductivity which is considered as random variable in the crack. A comparison with deterministic solution highlight the interest of the consideration of a stochastic approach in modeling properties uncertainty.

We note that the the power density has the same behavior as the electrical conductivity one

The resistance inquire more information about the presence of the default.

VI. CONCLUSION

A non-intrusive projection method is used to study non-destructive testing systems under Matlab environment. The electrical conductivity is considered as an essential physical property to characterize the defect area uncertainty.

References

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