Measurement of thickness of thin metal sheets by the $A_0$ Lamb mode at low frequency.

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Abstract—Industrial structures undergo significant stresses during their life and can destroy the environment. This can, in many cases, cause serious damage, often imperceptible. Several methods are used to detect and locate such early stage damage. Lamb waves, which are guided by both surfaces of a planar structure, are good candidates to inspect the plate in a non invasive manner. These waves have the advantage to travel long distances without significant attenuation in the direction of propagation, which allows control quickly, permanently and in real time, large areas of difficult access.

In this work, we present a method to measure the thickness of thin sheets using the propagation of the first anti-symmetric $A_0$ Lamb mode, generated by a piezoelectric transducer. The principle is based on the existing proportionality relationship between the group velocity and thickness of thin metal sheets, at low frequencies. We test this relation on various metallic sheets of thicknesses: 25; 75; 100; 160 and 200 microns.

Keywords—Lamb waves; group velocity; thickness; transducer.

I. INTRODUCTION

Planes, industrial and naval structures are generally subjected to constraints, bound to their functioning, and to climatic conditions, from where an austere corrosion which can attack the metal. This phenomenon can cause an important decrease of the thickness. It can turn out dangerous, because the variation of thickness weakens structure. To control, in a non-destructive way, these plate structure and to avoid accidents, use of guided waves, is a very interesting solution. When the structure is a thin sheet, it’s possible to generate Lamb waves. These waves are multimodal and often dispersive. Several authors published articles treating about the use of Lamb waves to detect the thickness variation in plates [1, 2]. All these methods give good results to detect variations thickness. However, set apart the laser method [3], they do not allow measuring with accuracy this variation. This paper describes a method which allows determining, by the measure of the group velocity, immediately, the thickness of a sheet and the variation of depth creates by a defect. At very low frequency, only two Lamb modes can be generated $S_0$ and $A_0$ [4]. However, only group velocity of the antisymmetric Lamb mode $A_0$ is proportional to the thickness of the plate.

This property is verified firstly theoretically from dispersions Lamb equations then experimentally for many plates of different thicknesses.

II. MAIN FEATURES OF GUIDED ELASTIC WAVES IN A PLATE

Lamb waves are guided waves induced by the free surfaces of the plate, and propagate along the planar structure. The peculiarity of Lamb waves is that the propagation velocity varies depending on the product frequency thickness. The Lamb waves may be symmetric or antisymmetric. In addition, each mode has several orders in certain frequency ranges. The Lamb waves propagation velocity is calculated using the Rayleigh–Lamb “1” derived from the displacement potential of an elastic body and from Newton’s equations

$$\frac{\omega^4}{V_T^4} = 4k^2q^2\left[1 - \frac{p\tan(ph + \alpha)}{q\tan(qh + \alpha)}\right]$$ (1)

With $\alpha = (0, \pi/2)$ for symmetric mode and antisymmetric mode respectively and, where the parameters $p$ and $q$ are defined as follows:

$$p^2 = \frac{\omega^2}{V_L^2} - k^2 \text{ and } q^2 = \frac{\omega^2}{V_T^2} - k^2$$

$\omega$ is angular frequency, $k$ is wave number, and where $V_L$ and $V_T$ are the propagation velocities of bulk wave. The thickness is equal to $2h = d$ (thickness).

Given the mechanical displacements, two types of motion can propagate in a plate: one symmetric (longitudinal) and the other antisymmetric (flexural). One problem associated with Lamb waves is the coexistence of many undesired modes in the plate. All Lamb modes, except for the lowest-order symmetric ($S_0$) and antisymmetric ($A_0$) modes have lower cut off frequencies, from “1”.

In the other hand, the lowest order group velocity dispersion curves can be derived from the phase velocity curves, the curves are plotted in Fig. 1.
Fig. 1. Group and phase velocities for the fundamental modes of Lamb waves in metal sheet, having a thickness of 200 µm, in the case where $V_L = 5032.46$ m/s and $V_T = 2723.09$ m/s.

III. RELATION BETWEEN $A_0$ MODE VELOCITY AND SHEET THICKNESS AT VERY LOW FREQUENCIES

A. Phase velocity at very low frequencies

At very low frequencies, the phase velocity varies with thickness. To highlight this relationship, the dispersion equation of Rayleigh-Lamb is simplified [5].

$$\left(k^2 - q^2 \right)^2 = \omega^4 / V_T^4$$ (2)

and

$$\left(k^2 - q^2 \right)^2 = (k^2 + q^2)^2 - 4k^2 q^2 = \frac{\omega^4}{V_T^4} - 4k^2 q^2$$

The equation (1) becomes

$$\frac{\omega^4}{V_T^4} = 4k^2 q^2 \left[1 - q \tan(qh + \alpha) \right]$$ (3)

At very low frequencies, we are able to limit the development of equation (2) in vicinity to 0.

For $A_0$ mode when $k$ tends towards 0, $\omega$ also tends towards 0.

The equation (2) becomes:

for $\alpha = \pi/2$ (antisymmetric mode)

$$\frac{\omega^4}{V_T^4} = 4k^2 q^2 \left[1 - q \cot(g(ph)) \right]$$ (4)

By limiting the development of $\cot(g(ph))$ at order two and for $ph << 1$ and k.h << 1, we obtain

$$\frac{\omega^4}{V_T^4} = \frac{4k^2 q^2 q^2}{3} (p^2 - q^2)$$ (5)

and

$$V_p = \frac{\omega}{k} = \left(\frac{2\pi V_p^2 \sqrt{\frac{1}{3}}}{h} \right)^{1/2}$$ (6)

Here $V_p$ is the phase velocity and $V_s$ is the sheet velocity: when the product fh becomes small the velocity of the symmetric mode $S_0$ tends to $V_p$ (the wavelength becomes large compared to the sheet thickness 2h) defined by:

$$V_p = 2V_T \left(1 - \frac{V_T^2}{V_L^2} \right)^{1/2}$$ (7)

and

$$V_p = \frac{\omega}{k} = 2\pi f h$$ (8)

So we get

$$V_p = V_p \left(\frac{2kh}{2\sqrt{3}} \right)$$ (9)

Effectively in the Equation (9), the phase velocity is proportional to the product wave number thickness (2kh) which depends on the used frequency. It is easier to obtain group velocity that phase velocity so it would be more interesting to find a relation between group velocity and the thickness.

B. Group velocity at very low frequencies

The other major characteristic of Lamb wave is dispersion which means that the amplitude and duration of Lamb wave forms changes with distance along the propagation path. It is caused by the difference between the group and phase velocities. $A_0$ mode is dispersive at low frequencies. The effect of dispersion is that the energy in a wave-packet propagates at different speeds depending on its frequency. This manifests itself by spreading of the wave-packet in space and time as it propagates through a structure. The theoretically group velocity at very low frequencies is derived from “11”:

$$V_g = \frac{\partial\omega}{\partial k} \approx \frac{\partial(V_p \times k)}{\partial k} \approx \frac{2khV_p}{\sqrt{3}} = \frac{2(2f h^2\pi V_p)}{V_p \sqrt{3}}$$ (10)

The “11” is very interesting because, it’s allows to link the group velocity, which is determined experimentally, and sheet thickness. We note that at very low frequencies the group velocity is twice the phase velocity.

$$V_g \approx 2V_p$$ (11)

whence,

$$V_g^2 = 4\pi f d V_p / \sqrt{3}$$ (12)

and

$$d = V_g^2 \sqrt{\frac{1}{3}} (4\pi f) V_p$$ (13)

What precedes shows that it is possible to measure the sheet thickness, knowing the group velocity. We point out that the “13” is verified only at very low frequencies [5–8].

IV. EXPERIMENTAL SETUP

The experiments are carried out on thin metal sheets whose thicknesses are 25; 75; 100; 160 and 200 µm, according to the setup in Fig. 2. $A_0$ Lamb mode is generated by contact method, using piezoelectric transducer [9–11]. The emitter is driven by a pulse, from the pulse generator it is used to simultaneously trigger the numeric oscilloscope. We use one transducer (emitter/receiver) for generating and receiving waves Lamb. For this experiment we use two transducers, the first of central frequency 0.48 and the second 1.8 MHz. For good coupling between the metal sheet and the transducer, we use ultrasound gel.
The transducer is held by a support which allows to avoid charging the metal sheet (Fig. 2). It barely touches the metal sheet; it does not put pressure on it. It can move freely and without friction on the plate (Fig. 2). As a first step, we want to confirm that it is possible to generate the \( A_0 \) Lamb mode when the transducer is placed directly on the surface of the sheet, which, contrary to what is assumed in the theoretical study is not free, but placed on a wooden stand (on the Fig. 2, we note that the sheet remains horizontal despite the transducer that has no pressure on it) which may include an additional error in the thickness measurement. The pulse generator excites the transducer transmitter/receiver which receive the wave reflected by the free edge of the metal sheet. The signals obtained, allow us to measure the group velocity of the generated mode and by comparing this velocity with that obtained theoretically by dispersion curves, we can be sure about the nature of the mode. The signals obtained for the different sheets (Fig. 3) shows that the flexural \( A_0 \) Lamb mode is generated. For the sheet of 25 microns thickness it was used successively a 0.48 MHz transducer and another of 1.8 MHz. This, in order to show that, for a given sheet, when the frequency increases, the thickness measurement becomes less accurate. As frequency increases, the product \( f \times d \) increases (here from 0.012 to 0.045 MHz.mm). The signals obtained for the different sheets (Fig. 3) show that the flexural \( A_0 \) Lamb mode is generated. For all metal sheets (Fig. 3), we get a usable signal for which it is easy to measure the flight time which allows to calculate the group velocity knowing the distance traveled by the wave and therefore calculate the sheet thickness using the “13”.

**Fig. 3.** \( A_0 \) Lamb mode, on thin metal sheets whose thicknesses are (a) 25; (b) 75; (c) 100; (d) 160 and (e) 200 µm.

**V. EXPERIMENTAL RESULTS**

For different sheets, we generate the antisymmetric mode \( A_{0b} \), we measure the time of flight of this mode, knowing the length of the plate we deduce the \( A_0 \) mode group velocity. Equation (13) allows us, for \( V_p \) (sheet velocity; “7”) given, to calculate the sheet thickness \( d = 2 h \).
The experimental results are given in Table I. We observe that the results obtained by measurements using a palmer, are very close to those obtained by the A_0 Lamb method. The absolute error in thickness is the difference between the experimental value (A_0 Lamb method) and the value obtained with a micrometer (palmer), this variation ranges from 0.01 µm to 20 µm. The absolute error does not give a clear idea on the sensitivity of the measurement. We must study the relative uncertainty for each method for more accuracy. The accuracy or the absolute uncertainty got with the Lamb wave method is significantly better than the accuracy obtained with a micrometer in fact, the accuracy varies from 0.08% to 1.17% for the Lamb wave method and varies from 4% to 0.5% for the measurement with the micrometer (palmer). We can consider that these results are quite acceptable, despite the fact that the plate is not free, as assumed in the theoretical study; but under load. Moreover, it is clear that when the thickness is very thin, the linear relationship between the square of the group velocity and the product (f.d) is better and it gives better accuracy for the thickness measure.

### TABLE I. Summary table of results.

<table>
<thead>
<tr>
<th>(f.d) (MHz.mm)</th>
<th>Thickness measured with a palmer: d_p (µm) and accuracy ([(d_p - d)/d_p]×100) (%)</th>
<th>V_g (m/s)</th>
<th>Thickness determined by “13”: d + Δd (µm)</th>
<th>error Δ = d_p - d (µm)</th>
<th>Relative uncertainty (accuracy) (Δd/d_p) ×100 (%)</th>
<th>Experimental V_g (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.012</td>
<td>25 ± 1 [4%]</td>
<td>3941.10</td>
<td>25.03 ± 0.21</td>
<td>0.05</td>
<td>0.84 %</td>
<td>586.43</td>
</tr>
<tr>
<td>0.045</td>
<td>25 ± 1 [4%]</td>
<td>3941.10</td>
<td>25.33 ± 0.07</td>
<td>0.33</td>
<td>0.28 %</td>
<td>1142.19</td>
</tr>
<tr>
<td>0.036</td>
<td>75 ± 1 [1.33%]</td>
<td>4162.20</td>
<td>75.01 ± 0.06</td>
<td>0.01</td>
<td>0.08 %</td>
<td>1042.73</td>
</tr>
<tr>
<td>0.048</td>
<td>100 ± 1 [11%]</td>
<td>3972.50</td>
<td>99.47 ± 0.29</td>
<td>0.53</td>
<td>0.29 %</td>
<td>1173.07</td>
</tr>
<tr>
<td>0.077</td>
<td>160 ± 1 [0.6%]</td>
<td>4813.70</td>
<td>155.50 ± 1.52</td>
<td>5.0</td>
<td>0.97 %</td>
<td>1614.53</td>
</tr>
<tr>
<td>0.096</td>
<td>200 ± 1 [0.5%]</td>
<td>4580.0</td>
<td>180.60 ± 2.13</td>
<td>20</td>
<td>1.17 %</td>
<td>1697.23</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In this study, we showed that it was possible to measure the thickness of thin metal sheet, using a piezoelectric transducer to generate Lamb modes. A series of thin sheets, having a 150 mm long and 70 mm width, whose thickness ranges from 25 to 200 µm was tested. Measurements are more accurate when the thickness is thin, this being due to the fact that the linear relationship between the square of the group velocity and the product f.d is better. The results are very conclusive and show that the accuracy or the absolute uncertainty get with the Lamb wave method is significantly better than the accuracy obtained with a micrometer in fact, the accuracy varies from 0.08% to 1.17% for the Lamb wave method and varies from 4% to 0.5% for the measurement with the micrometer (palmer). The feasibility has been verified, we plan to repeat the experiment with a transducer center frequency of a few kHz, the results will, certainly, be better since the linear relationship between the square of the group velocity and the product f.d, will be checked precisely. This sensitivity of the A_0 mode can be exploited for the detection and/or estimation of the thickness loss in the case of oxidation or corrosion.

**References**