Theoretical investigation of damage detection in composite structures using harmonic response technique

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Abstract*— Nowadays the fiber-reinforced composite materials have used widely in the construction of aircraft structures; these structures are subjected to different excitations which may cause serious failure. For this reason our research presents a technique aimed to identify the damage in fiber-reinforced composite beam subjected to harmonic load using analytical model. The presence of a crack in structural elements leads to an energy concentration near to the crack region and introduce a local flexibility which affects its dynamical characteristic. Due to this fact, the harmonic response technique assumed to be reasonable technique for damage identification in cracked beam through the evaluating of its dynamic response to harmonic excitation. The discrete spring model has been used for modeling the crack, including the stress intensity factors, in which the composite beam is assumed to be divided into two sub-beams at the crack position, connected together by the additional equivalent spring. Euler-Bernoulli beam and the modal expansion theories are used in this study for solving the differential equation related to forced vibration. A parametric study has been carried out in order to investigate the influence, of crack depth and crack location on the transverse displacement of cracked composite beam under harmonic excitations. The obtained results show that the response amplitudes of a cracked beam changes with the varying of crack depth and location, and therefore, it can be used as a crack detection criterion. The dynamical behavior of composite beam with a lower flexural rigidity is more sensitive to presence of a crack. The vibration amplitudes are more sensitive when the crack depth increases. As a consequence, the evaluation of dynamical response of cracked beam subjected to moving harmonic excitation can be used as an appropriate technique for damage detection in composite structures.

***Keywords*—Composite beam; Damage detection; harmonic excitation; Transverse displacement;**

I. INTRODUCTION

The Fiber Reinforced composite materials have been attracted many engineering industry, such as aerospace, naval construction and civil building. The increase of their using in many fields, due mainly to various properties, such as ,high rigidity, resistance to environmental conditions and high fatigue strength.

The existing crack in structural elements may affect their mechanical behavior and lead to damage failures. Due to this fact, many researchers have focused their investigations in the enrichment of mechanical proprieties of these structures. The presence of cracks can be introducing a local flexibility which has an influence on their dynamic characteristics. However, the evaluation of dynamical behavior of cracked structures can be used as indicator in the predicting of a crack. This research topic has been tackled in many papers.

The deriving of crack formulation was based on fracture mechanic theory including the stress intensity factors which considered as important parameters for identifying the flexibility matrix. Krawczuk [7] has determined the flexibility coefficients matrix of composite beam containing a single crack. According to literature, it can be found a range of analytical and numerical methods for studying the influence of existing cracks on dynamic characteristic of composite beam. Concerning the analytical methods, the works have done by Wang et al.[10], focused in the developing of a new mathematical model for evaluating the natural frequencies and theirs corresponding mode shape of unidirectional cracked composite beam under coupled flexural–torsion vibrations .Semi-analytical method titled 'Generalized Differential Quadrature', used by Daneshmehr et al. [4] for free vibration analysis of cracked composite beam, based on first order shear theory. Ghoneam [5] have proposed a new analytical model to study the effect of fiber orientation and various boundary conditions in dynamic behavior of cracked and uncracked laminate composite beam. A transfer matrix method related to state vector has been used by Mostafa [9] for free vibration analysis and crack identification of stepped beams with multiple edge cracks and different boundary conditions.

Several researchers have interested in the using of numerical method like finite element method, for example. Krawczuk [8] have proposed new finite element for identification the dynamical characteristic of cracked composite beam. Behera et al.[3] used finite element method for free vibration of laminate composite beam with a transverse crack, based on first shear deformation. Kisa [6], studied the effect of crack on natural frequencies and their corresponding mode shape of a cantilever composite beam, using finite element associated with component mode synthesis methods. Other authors interested in the using of finite element analysis by means a numerical code[1] investigated the variations in the eigen-nature of cracked composite beams due to different crack depths and locations using numerical and experimental investigation.

In last decade years, the dynamic response of a cracked beam subjected to moving load or mass has been studied by many authors. Among of them. Sadettin Orhan [12] has used the forced vibration response of a cracked beam for identifying the crack depth and location. He conclude that the forced vibration is better than free vibration in the detection of cracks. Pugno [11] proposed a new technique for evaluating the dynamic response of a beam with several breathing cracks perpendicular to its axis and subjected to harmonic excitation. Sinha JK et al. [13] used experimental and numerical methods for simulating a crack of free-free beam. Lin HP et al.[9] used Transfer matrix method and modal expansion theory for studying the forced responses of cracked beam

This present paper in devoted for studying the effect of an edge crack on the flexural vibration of unidirectional composite beam. A Parametric study has carried out in order to investigate the influence of fiber orientation, crack depth and crack location on the natural frequencies and theirs corresponding shape modes. The dynamic analysis of cracked composite beam used as technique for prediction the fatigue damage.

II. CRACK FORMULATION THEORY

The presence of a crack in structure leads to energy concentration in the region near to the crack and introduce a local flexibility which affect the dynamic characteristics of these structures, considering a composite beam containing a transversal edge crack, shown in Fig. 1, the beam is subjected to pure bending load P.

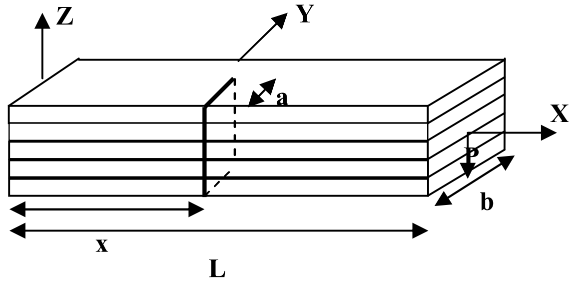

Fig 1. Geometry of cracked composite beam

Based on linear fracture mechanic theory, the crack formulation have been derived in which, the flexibility coefficients are expressed as a function of stress intensity factors and crack geometry.

The additional displacement due to presence of a crack in rectangular cross section, under the acting of load P , determined by the Castigliano's theorem

$$u_i = \frac{\partial U}{\partial P_i} \tag{1}$$

where U is the strain energy of a cracked structure, defined as follows :

$$U = \int_A \left(D_1 \sum_{i=1}^{i=N} K_{Ii}^2 + D_{12} \sum_{i=1}^{i=N} K_{Ii} \sum_{i=1}^{i=N} K_{IIi} + D_2 \sum_{i=1}^{i=N} K_{IIIi}^2 \right) dA \tag{2}$$

where A denots the cracked surface, KIn, KIIn, et KIIIn are the stress intensity factors for fracture mode I, II and III, respectively, these factors were used to evaluate the stress field around the cracks region , D1, D2 and D12 are coefficients depending on the materials parameters, are taken from [7]

The stress intensity factors for cracked composite beam, given as [2]

$$K_{ij} = \sigma_n \sqrt{\pi a} Y_j(\zeta) F_{jn}(a/b) \tag{3}$$

where σn is the stress of cracked area which relating to corresponding fracture mode, is the the correction function dependent in geometry , $Y_j(\zeta)$ is the correction factor for the anisotropic material [10], a is the crack depth and, L and b are the length and the breadth of beam respectively. The stress intensity factor corresponding to cracked beam under bending loading about z-axis , given as follows

$$K_I = \sigma_4 \sqrt{\pi a} Y_1(\zeta) F_1(a/b), \sigma_4 = \frac{12P}{bh^3} z; \tag{4}$$

where

$$F_1(a/b) = \sqrt{\frac{\tan\lambda}{\lambda}} \left[0.752 + 2.02(a/b) + 0.37(1-\sin\lambda)^3 \right] / \cos\lambda, \lambda = \frac{\pi a}{2b}$$

and

$$Y_1(\zeta) = 1 + 0.1(\zeta - 1) - 0.016(\zeta - 1)^2 + 0.002(\zeta - 1)^3;$$

The flexibility coefficients of cracked composite beam under loading are expressed in the following form

$$c_{ij} = \frac{\partial^2}{\partial P_i \partial P_j} \int_{-h/2}^{h/2} \int_0^a J(\alpha) d\alpha dz \tag{5}$$

where

$J(\alpha) = \partial U / \partial \alpha$ is the elastic strain energy release rate J, a is the crack depth .

The coefficient flexibly parameter used in the case of pure bending load, considered herein, is c_{44} and given as [10].

$$c_{44} = \frac{24\pi D_1}{b^2 h^3} \int_0^a \alpha \left[F_1(\alpha/b) \right]^2 d\alpha; \tag{6}$$

III EQUATION OF MOTION FOR FLEXURAL VIBRATION

The free flexural vibration of an Euler-Bernoulli beam with constant cross-section is expressed in the following differential equation

$$EI\frac{\partial^4 h}{\partial y^4} - m\frac{\partial^2 h}{\partial t^2} = 0 \tag{7}$$

where EI represents the flexural rigidity of unidirectional composite beam, m is the mass per unit length, given as [7].

$$EI = b\left(D_{22} - \frac{D_{12}^2}{D_{11}}\right); \tag{8}$$

$$m = b\rho h; \tag{9}$$

Bu using separation of variable $h(y,t) = H(y)e^{i\omega t}$, the Eq. 8 yield

$$EIH^{iv} - m\omega^2 H = 0 \tag{10}$$

where $\lambda = \left(m\omega^2 L^4/EI\right)^{1/4}$,

The general solution of the above equation is

$$H(x) = A_{k1}\cosh\lambda x + A_{k2}\sinh\lambda x + A_{k3}\cos\lambda x + A_{k4}\sin\lambda x \tag{11}$$

The expressions for cross-sectional rotation Θ (x),the bending moment M (ξ) and the shear force S (x) become

$$\Theta(x) = (1/L)\left[A_{k1}\lambda\sinh\lambda x + A_{k2}\eta\cosh\lambda x - A_{k3}\eta\sin\lambda x + A_{k4}\eta\cos\lambda x\right], \tag{12}$$

$$M(x) = \left(EI/L^2\right)\begin{bmatrix} A_{k1}\lambda^2\cosh\lambda x + A_{k2}\lambda^2\sinh\lambda x - A_{k3}\lambda^2\cos\lambda x - \\ A_{k4}\lambda^2\sin\lambda x \end{bmatrix}, \tag{13}$$

$$S(x) = -\left(EI/L^3\right)\begin{bmatrix} A_{k1}\lambda^3\sinh\lambda x + A_{k2}\lambda^3\cosh\lambda x + A_{k3}\lambda^3\sin\lambda x - \\ A_{k4}\lambda^3\cos\lambda x \end{bmatrix}. \tag{14}$$

The cracked composite is divided in two sub-beams at cracked section. Thus, the continuity conditions for an edge crack located at distance $x = x_c$ can be expressed as [10]

$$M_i(x_c) = M_{i+1}(x_c) \tag{15}$$

$$S_i(x_c) = S_{i+1}(x_c) \tag{16}$$

$$H_i(x_c) = H_{i+1}(x_c) \tag{17}$$

Moreover, a discontinuity cross-sectional rotation due to an existing crack can be expressed as follows

$$\Theta_i(x_c) = \Theta_{i+1}(x_c) - c_{44}M_i(x_c) \tag{18}$$

where c_{44} is the crack-sectional flexibility coefficient parameter , using in the case of cracked beam under bending load in vertical direction. The subscript (i) and (i+1) in Eqs 15,16,17 and 18 refers to left and right of sub-beam at cracked section.

The boundary conditions of clamped composite beam are :

- at fixed end

$$H_i(0) = \Theta_i(0) = 0; \tag{19}$$

- at free end

$$M_{i+1}(1) = S_i(1) = 0; \tag{20}$$

Substitution of Eqs 11- 14 into Eqs 15-20 will yield the characteristic equation of the cracked composite beam.

$$[B]\{A\} = \{0\} \tag{21}$$

where

$\{A\} = \{A_{11}, A_{12}, A_{13}, A_{14}, A_{21}, A_{22}, A_{23}, A_{24}\}^T$ is a vector composed of 8 unknowns and $[B]$ represents a characteristic matrix given as function of natural frequency

III. FORCED FLEXURAL VIBRATION

Consider a composite beam clamped at left end and free at right end, having length L, height t and width b, containing an edge crack of length, a , with the force $f(t)$ acting at the free end, as shown in Fig. 2 . The composite beam considered herein is made of unidirectional graphite fiber-reinforced, where its physical and geometrical characteristics are chosen similarly to [7].

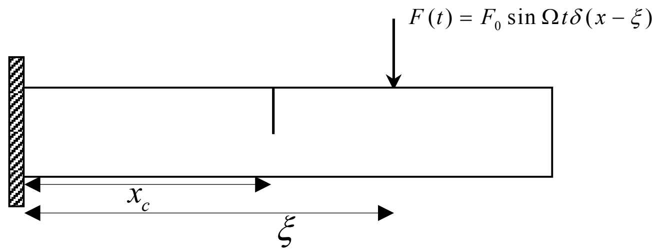

Fig.2. Cracked composite beam subjected to concentrated harmonic force.

The equation of motion relating to the forced flexural vibration of a beam is given as

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 \mathrm{w}(x,t)}{\partial x^2}\right] + \rho A(x)\frac{\partial^2 \mathrm{w}(x,t)}{\partial t^2} = f(x,t) \tag{22}$$

Using the modal expansion theory, the solution of Eq. (22) is assumed to be a linear combination of the normal modes of the beam as

$$\mathrm{w}(x,t) = \sum_{i=1}^{\infty} W_i(x)q_i(t) \tag{23}$$

where $W_i(x)$ are the normal modes should be determined by using the boundary conditions of the beam

$$\frac{d^2}{dx^2}\left[EI(x)\frac{d^2W(x)}{dx^2}\right]-\rho A(x)\omega_i^2 W_i(x)=0 \qquad (24)$$

and $q_i(t)$ are the generalized coordinates. Using Eq. (22), Eq. (23) can be expressed as

$$\sum_{i=1}^{\infty}\frac{d^2}{dx^2}\left[EI(x)\frac{d^2W_i(x)}{dx^2}\right]q_i(t)+\rho A(x)\sum_{i=1}^{\infty}W_i(x)\frac{d^2q_i(t)}{dt^2}=f(x,t) \qquad (25)$$

Using Eq. (24), Eq. (25) can be rewritten as

$$\rho A(x)\sum_{i=1}^{\infty}\omega_i^2 W_i(x)q_i(t)+\rho A(x)\rho A(x)\sum_{i=1}^{\infty}W_i(x)\frac{d^2q_i(t)}{dt^2}=f(x,t) \qquad (26)$$

Then, we multiply the equation of motion Eq. (26) by the normal modes w (x), and integrating from 0 to l, we obtaine

$$\sum_{i=1}^{\infty}q_i(t)l\int_0^l \rho A(x)\omega_i^2 W_j(x)W_i(x)dx+ \sum_{i=1}^{\infty}\frac{d^2q_i(t)}{dt^2}\int_0^l \rho A(x)W_j(x)W_i(x)dx=\int_0^l W_j(x)f(x,t)dx \qquad (27)$$

Using modes orthogonality relations foe expressing the equation of q_i (t) ,yields

$$\frac{d^2q_i(t)}{dt^2}+\omega_i^2 q_i(t)=Q_i(t) \qquad (28)$$

where Qi (t) is the generalized force corresponding to the ith mode given by

$$Q_i(t)=F_0\int_0^l W_i(x)\sin\Omega t\,\delta(x-\xi)\,dx=F_0 W_i(\xi)\sin\Omega t \qquad (29)$$

The complete solution of Eq. (28) can be expressed as follows

$$q_i(t)=\frac{1}{\omega_i}\int_0^t Q_i(\tau)\sin\omega_i(t-\tau)d\tau= \frac{F_0}{\omega_i}W_i(\xi)\frac{1}{1-\Omega^2/\omega_i^2}\left(\sin\Omega t-\frac{\Omega}{\omega_i}\sin\omega_i t\right) \qquad (30)$$

Finaly ,the response of the beam can be expressed as

$$w(x,t)=F_0\sum_{i=1}^{\infty}\frac{W_i(x)W_i(\xi)}{\omega_i^2-\Omega^2}\left(\sin\Omega t-\frac{\Omega}{\omega_i}\sin\omega_i t\right) \qquad (31)$$

where $W_i(x)$ and $W_i(\xi)$ correspond to the normalized normal modes.

IVNUMERICAL EXAMPLE AND DISCUSSION

Consider a cracked composite beam clamped at left end and free at right end, containing an edge crack located at a distance x_c from the left end, as shown in Fig. 2 . The composite beam considered herein is made of unidirectional graphite fiber-reinforced, where its physical and geometrical characteristics are chosen similarly to [7].

The material properties of the graphite fiber-reinforced composite beam are given in Table. 1. The geometrical characteristics, the length (L), height (t) and width (b) of the composite beam, are taken as 0.5m, 0.005m and 0.1m respectively

Table 1 : The material properties of the graphite fiber-reinforced composite beam

E_f (GPa)	E_m (GPa)	G_f (GPa)	G_m (GPa)	ρ_f (Kg/m^3)	ρ_m (Kg/m^3)	ν_f	ν_m
275.6	2.76	114.8	1.036	1900	1600	0.2	0.33

the variation of fiber angle from 0° to 90°, when the fiber angle is greater than 45°, these changes become more sensitive, this due to fact,the change in flexural rigidity which is a function of fiber angle. In Fig. 3, it is noticeable also the increasing in crack depth led to reduction in fundemntal natural frequencies.

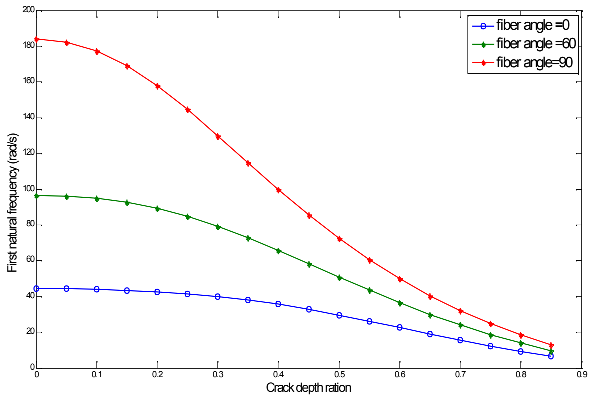

Fig .3 Effect of crack depth on fundamental natural frequency of the cracked composite beam for different angle of fiber

The variation of first natural frequency of cracked composite beam for range of crack position is illustrated in Fig.4. As observed from results that, these changes due to variation of fiber angle for a crack located at distance (x/L=0.3) from the fixed end of composite beam is lower compared to those located at 0.4 and 0.5, therefore , the crack position has also affect the variation of natural frequencies.

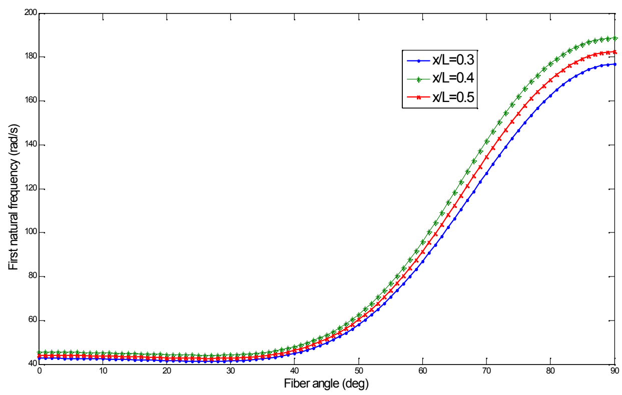

Fig. 4. Effect of angle of fiber on fundamental natural frequency of the cracked composite beam for different crack location x/L=0.3, 0.4, 0.6

In Fig. 5 demonstrates the variation of fundamental natural frequency as a function of crack depth ration, the increasing of crack depth reduces the first natural frequency. the results show also that these variations of natural frequency to crack position at a distance (x/L=0.2) from the fixed are less sensitive compared to other cracks positions (x/L=0.4, 0.6).

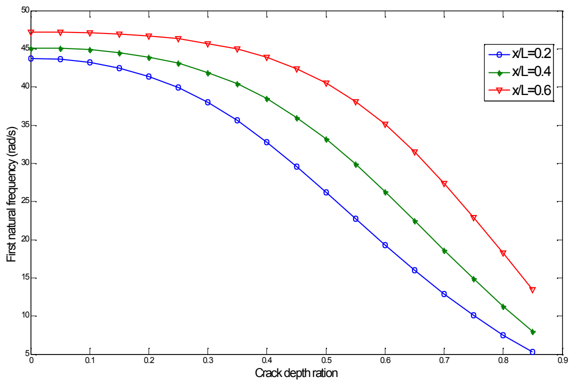

Fig. 5. Effect of crack depth on fundamental natural frequency of the cracked composite beam for different crack location ration x/L=0.2, 0.4, 0.6

The numerical results in this following section have been adopted for evaluating the effect of fiber angle , crack depth ration and crack location on the dynamic response of a cracked composite beam, subjected to harmonic excitations.
The influence of the fiber angle on the amplitudes of transverse displacement of cracked composite beam is shown in Fig 6, where the fiber volume fraction considered herein is (V=0.5) and the crack is located at a distance (x/L=0.3). Based on graphically results shown in this figure, it is observed that the change in the amplitudes of transverse displacement is caused by the change in fiber angle. However, the dynamic response decreases with an increase in flexural rigidity which is a function of fiber angle.

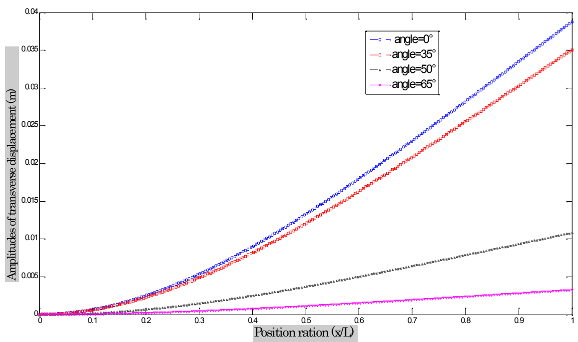

Fig .6. Amplitudes of transverse displacement of composite beam for different fiber angle .

Fig 7 shows the dynamic response of cracked composite beam for various crack depth (a/b=0.2, 0.4, 0.6, 0.8), fiber angle assumed to be zero degree and the crack is located at a distance (x/L=0.3). It is seen that the transverse displacement is influenced by the presence of crack furthermore the increasing in the amplitudes is much more significantly by the increasing of the crack depth. In fact this is because the reduction of flexural rigidity of composite beam.

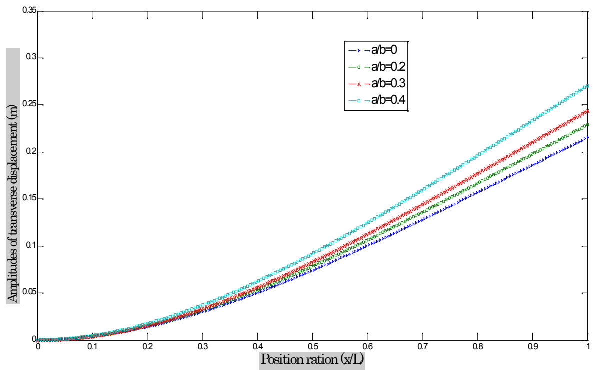

Fig. 7. Effect of crack depth on the amplitudes of transverse displacement of composite beam

The variation of transverse displacement amplitudes for range of crack position is illustrated in Fig. 8. As observed from results that these changes in vibration amplitudes is due to variation in crack positions. Therefore, the dynamic response is much more significantly influenced in the case of crack which is located at distance near to a fixed end of beam, due to fact, that the bending moment is higher at this position.

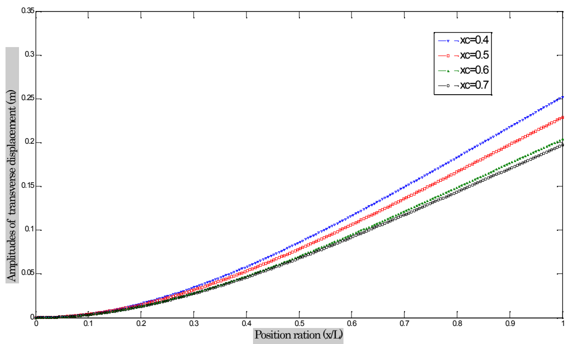

Fig. 8. Effect of crack position on the amplitudes of transverse displacement of composite beam

CONCLUSION

The forced flexural vibration of fiber reinforced composite containing an edge crack, based on Euler-Bernoulli theory is analytically studied using the concept of flexibility matrix for modeling the crack and the modal expansion theory for solving the forced response equation. The obtained results show the influence of several parameters, such as fiber angle, crack depth ration and crack positions on the amplitudes of transverse displacement; however,

The dynamical behavior of composite beam with a lower flexural rigidity is more sensitive to presence of a crack. The vibration amplitudes are more sensitive when the crack depth increases. In addition to these numerical results, the dynamic response is more significantly affected when the crack located near to fixed end.

As a consequence, the evaluation of dynamical response of cracked beam subjected to harmonic excitation can be used as an appropriate technique for damage detection in composites structures.

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